MELLIFICIUM MENSION

OR, THE

Marrow *of* Meaſuring.

WHEREIN

A New and Ready Way is ſhewn how to Meaſure *Glazing*, *Painting*, *Plaiſtering*, *Maſonry*, *Joyners*, *Carpenters*, and *Bricklayers* Works.

AS ALSO,

The *Meaſuring* of LAND, and all other *Superficies*, and *Solids*, by *Vulgar Arithmetick*, without reducing the *Integers* into the leaſt Denomination, giving the Content of any *Superficie* or *Solid*, con[illegible] *Feet*, *Inches*, and *Parts of Inches*, in a Fourth [illegible] *Time* and *Labour* required by the uſual Way in *Vulgar Arithmetick*.

TOGETHER

With ſome CHOICE PRINCIPLES and PROBLEMS of *GEOMETRY* conducing thereto.

To which is Added,

An APPENDIX, left under the AUTHOR's own Hand.

The CONTENTS *of which, ſee in the next Page.*

Illuſtrated with Copper CUTS.

The like not heretofore Publiſhed.

The Fourth Edition, with Additions.

By VENTERUS MANDEY.

No[illegible] ſed Quales.

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IN this *Edition* is added an APPENDIX. Wherein,

I. The Errors and Diſagreements of ſeveral Meaſurers, in meaſuring the Bricklayers Work of Chimneys of all Sorts, are reformed: Which Miſtakes and Errors are the more pardonable, if we conſider, that the moſt Part of the Work relating to Chimneys, is inviſible, or hid.

II. Rules are laid down for the Meaſuring of Arches and Vaults, whether they are Semicircles, Segments of Circles, or Ellipſes, together with their Butments.

III. The Menſuration of ſome Surfaces and Solids, which were not inſerted in the firſt and ſecond Editions.

IV. Two TABLES, one whereby to determine the Price of one, or any Number of Feet of Brickwork, at any Price *per* Rod: The other, to find the Perpendicular of any Gable End, by having its Baſe.

V. Some Alterations in the Method of Meaſuring are propoſed, as being more agreeable to Truth than the Ways heretofore practiſed.

VI. The whole Work is corrected and amended, by omitting ſome Things, and adding others, where Neceſſity required it.

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THE AUTHOR'S EPISTLE TO THE READER.

Courteous Reader,

THE Subject of the enſuing Treatiſe, is the Science of Meaſuring in a new and brief Method by vulgar Arithmetick, wherein the laborious and troubleſome Part of reducing the Feet and Inches into the leaſt Denomination, *i. e.* into Parts of Inches, is omitted, and a brief and ſpeedy Way directed for the caſting up any Dimenſion whatſoever.

Which Way of Meaſuring, conſidering that Workmens Rulers are not divided decimally, and that moſt Dimenſions are taken by a Ruler divided into Feet, Inches, Half Inches, and Quarters, and for the moſt Part, the Contents are required in the ſame Kind; I ſay, Conſidering this, it cometh very little ſhort of the Decimal Way of Meaſuring.

For if Dimenſions be given in Feet, Inches, and Halfs, or Quarters of Inches (as moſt uſually they are) and the Contents required in the ſame Method, then I look upon the reducing thoſe Dimenſions into Decimals, and after the Contents are found in Decimals, the reducing them again into Feet, Inches, and Half Inches, *&c.* to take up more Time than this Way which is taught in the enſuing Treatiſe.

Beſides, many Men can multiply and divide by vulgar Arithmetick, which do not underſtand Decimals; for whoſe Sakes chiefly this is written.

I conceive every Book meets with many critical Cenſures, and I doubt not, but this will partake with the reſt; and therefore it might (perchance) be expected, that I ſhould excuſe myſelf for whatſoever any Man ſhall be pleaſed to object againſt in it, which I ſhall neglect, only deſiring the judicious Reader to paſs by ſome ſmall Overſights, which, perhaps, there may be crept into the enſuing Work, as knowing that all Mens Doings are ſubject to Error: But as for groſs Faults, I think there are none; for I have been as careful as I could, both in writing, and likewiſe in correcting the Printed Sheets in this Edition, conſidering my want of Time, occaſioned by my other Employment.

To perſuade any one to the Study of this Science, would be a Folly, ſince the Ignorant (which are blind) cannot judge truly; and to him that already underſtands it, the Labour would be uſeleſs and unprofitable; and to the Averſe and Careleſs, it would be like the caſting of Pearls before Swine; the exquiſite Knowledge whereof, cannot be attained by ſuch prejudicated Perſons, altho' to many that are judicious, the Uſefulneſs hereof is already ſufficiently known; and to thoſe induſtrious Perſons which are yet to ſeek in the Knowledge of this Subject, and deſire to be informed, this Treatiſe will fully anſwer their Expectations.

In the following Treatiſe, I have endeavoured to proceed methodically, and have, to my Knowledge, omitted nothing, which might tend to the making of a Man an expert Meaſurer; in order to which, there are three Books of *Geometry* inſerted, the firſt containing the Rudiments thereof, and the ſecond and third containing choice Problems; which two laſt, I tranſlated from *Latin* Copies, wherein I have endeavoured to render the Meaning of the Author as plain as the Work would permit, and have amended ſome Miſtakes (I ſuppoſe) of the Printer of the *Latin* Copies: I have likewiſe explained the Meaning of ſome difficult Terms, and alſo added a new Diagram belonging to *Chap.* 6. *Lib.* 3. *Fig.* 1. which was wanting in the Copy.

So that the whole Treatiſe conſiſts of ſix Books, a general Account whereof follows.

The firſt Book of this Treatiſe contains the Rudiments of *Geometry*, conſiſting of ſuch Definitions and Propoſitions, as are meet to be known to any Man that intends the Science of Meaſuring, and is an Introduction to the other five Books; in which firſt Book (perhaps) ſome few of the Definitions may ſeem ſtrange, as not agreeing with the Definitions of ſome others, yet, in my Opinion, agreeable both to Reaſon and Truth. There is likewiſe added, the Deſcription of Ovals to any Length and Breadth, with a Pair of Compaſſes; and alſo a Digreſſion concerning Ovaller Arches.

The ſecond Book, being tranſlated from a *Latin* Author, contains a Garden of *Geometrical Roſes*, conſiſting of choice Propoſitions in *Geometry*, wherein a new Way is ſhewn of cutting right Lines in extream and mean Proportion; alſo the Diviſion of Angles, and finding a ſtrait Line, in Length equal to a Circular; as alſo to find the Centre of Gravity of a Semicircle or Quadrant, with ſeveral other Things not heretofore publiſhed in *Engliſh*, that I know of.

The third Book, being likewiſe tranſlated from the ſame Author, contains ſome Principles and Problems in *Geometry*, formerly thought deſperate, now briefly explained in *Engliſh*, conſiſting of the Subject, Principles, and Method of the *Mathematicks*, and of *Algebraical* Operations; likewiſe of a new Method of treating of Solids and their Superficies, by the efficient Cauſes, with ſeveral other Things.

The fourth Book contains the Science of *Menſuration*, ſhewing how to meaſure all kinds of Works relating to Building, to wit, *Carpenters*, *Glaſiers*, *Painters*, *Plaiſterers*, *Maſons*, and *Bricklayers* Works, *&c.* the Contents whereof may be ſeen more at large in the following Table.

The fifth Book contains the meaſuring of *ſuperficial Plains*, wherein is ſhewn how to meaſure *Triangles* of all kinds, *Rectangled Figures*, whoſe Sides are equal or unequal, *Circles*, *Ovals*, *Pyramids*, *Cones*, *&c.* In brief, it ſhews how to find the Content of moſt kind of ſuperficial Figures in uſe, whether they be regular or irregular, each Propoſition having a Figure or Diagram belonging to it, for the more eaſier underſtanding how to reſolve it.

This fifth Book alſo ſhews how to meaſure Land lying in any Form, and how to reduce Cuſtomary Meaſure into Statute Meaſure, and the contrary, *&c.*

The ſixth Book contains the Meaſuring of *ſolid Bodies*, ſhewing how to meaſure *Timber*, and *Stone*, &c. alſo how to find the ſolid Contents of *Pyramids*, *Cones*, *Cylinders*, *Spheres*, and other ſuch like Solids, each Propoſition having a Diagram (or Scheme) belonging to it, for the readier attaining the Reſolution of the Propoſition. In it is alſo contained the Science of *Gauging*, or meaſuring *Liquid Veſſels*, and how to find their Contents in Wine or Ale Gallons; and likewiſe how to find the Contents of *Brewers* Veſſels, Tuns, or Fats, in Gallons, and to reduce them into Barrels: All which, I have drawn into a Pocket Volume.

This Treatiſe thus finiſhed, I preſent thee with, deſiring thy friendly Cenſure and Acceptance of theſe my Labours; as alſo to paſs by ſuch Faults as may poſſibly have eſcaped the Preſs, or myſelf, which, I am certain, are but few: And in ſo doing, thou wilt oblige him, who is

A Friend to All, but more eſpecially to thoſe
that are Mathematically inclined,

V. M.

A TABLE

A TABLE

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O F

GEOMETRY *in General.*

GEOMETRY is a *Greek* Word, and imports only the Meaſuring of Land; and ſince ſeveral Parcels of Lands are 9 divided into various Forms, it behoves him, that would be eſteemed a Land-Meaſurer, to know how to meaſure all Kinds of ſuperficial Figures. But though the Word import no more but Land (or ſuperficial) Meaſuring; yet under the Name of GEOMETRY is compriſed the Meaſuring of all Kinds of Solids. This Science (according to Hiſtory) was firſt invented by the *Egyptians*; and the Cauſe which put them upon inventing it, was the Overflowing of the River *Nilus*, the greateſt and longeſt River in the World, which, when it overflowed, waſhed away their Banks and Land-Marks; and when the Waters were diſperſed, it was a difficult Matter for every Man to know his own Quantity of Land; inſomuch that it cauſed Quarrelling and Strife amongſt them, till at length, through the Induſtry of ſome ingenious Perſon or Perſons, this Science of GEOMETRY was found out, which put an End to their Quarrellings, and reſtored to every Man the ſame Quantity of Land after the Flood, that he had before.

IN brief, GEOMETRY is a Science, whereby the Quantities of Things not meaſured, are determined, by comparing them with other Quantities meaſured.

OF
GEOMETRY.

CHAP. I.

DEFINITIONS.

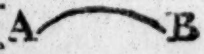
I. A POINT is a Body whoſe Quantity is not conſidered; if conſidered, is that which is not put to account in Demonſtration; and is made with the Point of the Compaſs, of a Pen or Pencil, or ſuch like, as the Point noted by A.

II. A Line is a Body whoſe Length is conſidered without its Breadth, and is made by the Motion of a Point from one Extreme to another.

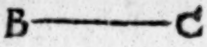
Extreme ſignifies the Beginning or End of a Line.

Of Lines are Three Sorts.

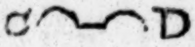
Firſt, Crooked or Circular. Secondly, Streight or Right. Thirdly, Mix'd.

1. Crooked or Circular Lines are thoſe, which have a Poſſibility of diducting or ſetting farther aſunder their Extremes, as the Line 

2. Streight or Right Lines are thoſe which have no Poſſibility of diducting or ſetting farther aſunder their Extremes; and therefore is the leaſt Length between the two Extremes. *Plato* defines a Right Line to be that, whoſe Extremes do ſhadow all the middle Parts, and is represented by the Line BC.

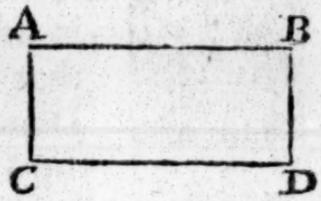


3. Mix'd Lines are compounded of Streight and Circular Lines, as the Line



III. A Superficie is that which hath Length and Breadth, the Thickneſs not being conſidered; and as a Line is produced by the Motion of a Point, ſo a Superficie is produced by the Motion of a Line ſuppoſed to move tranſverſly; that is to ſay, the Line AC being ſuppoſed to move ſide-ways to BD, it will produce a Superficies ABCD.

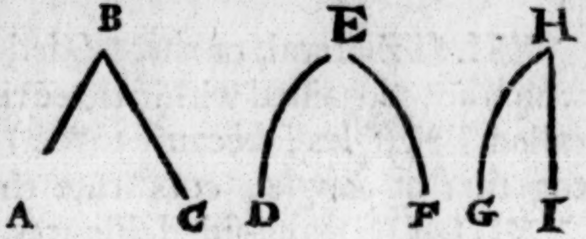
IV. The Extremes of a Superficie are Lines. But of a Circular Superficie, a Line is the Extreme.



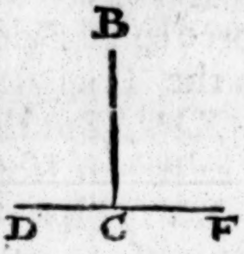
V. An Angle is an Inclination of two Lines, the one to the other, the one touching the other, and not lying ſtreight forth at length. And of Angles there are three Sorts, namely, Right-lined, Curve-lined, and Mix'd.

Note, When an Angle is mentioned by three Letters, the middlemoſt Letter ſignifies the Angle intended.

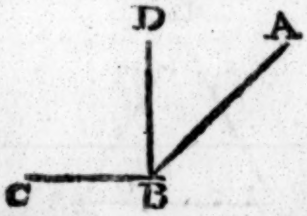
So ABC is a right lined Angle DEF is a curve lined Angle, and GHI is a mix'd Angle.



VI. When a right Line BC ſtanding upon a right Line DE, making the Angles on either ſide BCD, and BCE equal, they are called right Angles; and the right Line BC, is called a perpendicular Line to that upon which it is erected, *viz.* DE.



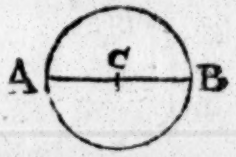
VII. An Angle is ſaid to be obtuſe or blunt, when it is greater than a right Angle; and acute or ſharp, when it is leſs than a right Angle: So the Angle ABC is an obtuſe Angle, and the Angle ABD is an acute Angle, and the Angle DBC is a right Angle.



VIII. A Circle is a plain Figure, comprehended by one Line, being generated by the Motion of a Compaſs, or other equivalent Means, wherein all right Lines drawn from the Centre to the Circumference, are of equal length. The Centre is a Point exactly in the Middle of the Circle.

IX. The Diameter of a Circle is a right Line, as AB drawn by the Centre C, and being terminated by the Circumference on either ſide, divides the Circle into two equal Parts.

X. The Semi-diameter is half the Diameter, as AC or CB.



XI. A Semicircle is one half of the whole Circle.

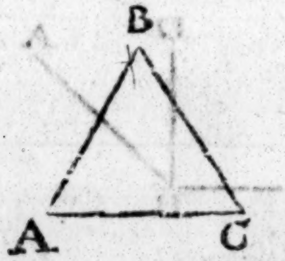
XII. Trilateral, or three-ſided Figures, are thoſe which are contained within three right Lines, and are called Triangles; becauſe three Lines being joined together at Angles, conſtitute three Angles.

XIII. Of three-ſided Figures, that which hath three equal Sides, is called an Equilateral Triangle, as the Triangle ABC.

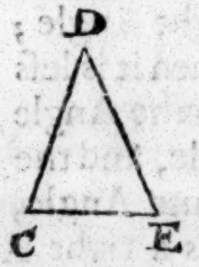
XIV. But that which hath two Sides alike equal, is called an Iſoſceles Triangle, as CDE.

XV. That Triangle, whoſe three Sides are unequal, is called a Scalenum, as EFG.

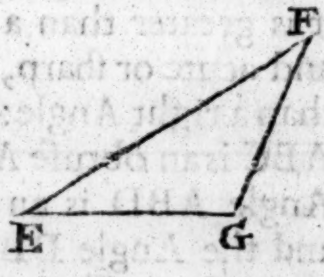
Equilateral.



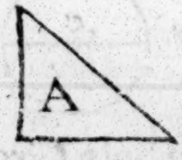
Iſoſceles.

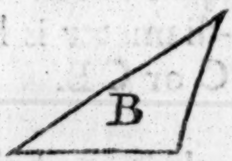


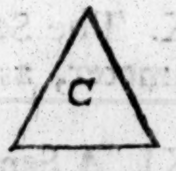
Scalenum.



XVI. There are alſo other Triangles, and are named according to the Quality of their Angles. A right angled Triangle, is that which hath one right Angle, as the Triangle A. An Ambligonium, or obtuſe-angled Triangle, is that which hath one Angle obtuſe, as the Triangle B. An Oxigonium, or acute angled Triangle, is that which hath three acute Angles, as the Triangle C.







XVII. In every Triangle, two of the Lines being taken for two ſides, the third Line remaining is called the Baſe, whether it be the lowermoſt Line of the Triangle, or not; ſo that any one of the three Lines, which incloſe a Triangle, may be taken for the Baſe.

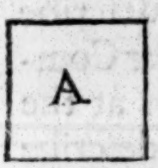
XVIII. Of Quadrilateral, or four-ſided Figures, there are ſeveral Sorts.

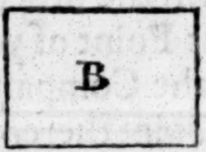
XIX. A Quadrate, or Square, is that which hath equal ſides, and right Angles, as the Figure A.

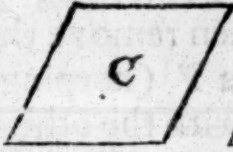
XX. An Oblong hath the oppoſite Sides alike, and right Angles, as the Figure B.

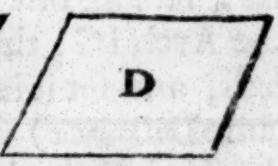
XXI. A Rhombus, or Diamond Figure, hath four equal Sides, and two Angles acute, and two obtuſe, as C.

XXII. A Rhomboides, hath oppoſite Sides, and oppoſite Angles alike, but it is neither equilateral nor right angled, as the Figure D.





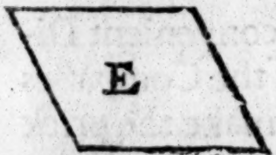


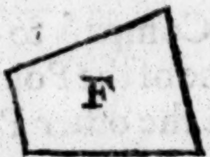


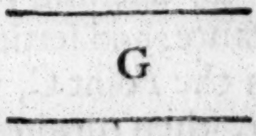
XXIII. A Paralallelogram is a four-ſided Figure, whoſe oppoſite Sides are parallel, as the Figure E.

XXIV. All other four-ſided Figures are called Trapezia's or Tables, as F, *&c.*

XXV. Parallel Lines are ſuch, which being in the ſame Superficie, if produced, will not meet, as G.







Thus much may ſerve for Definition; I proceed now to Practice.

CHAP. II.

Note, *That the Figures belonging to the following Propoſitions are in the next folded Page.*

PROP. I. Fig. I.

From a Point given in a ſtreight Line, to raiſe a Perpendicular, that ſhall cut the ſtreight Line at right Angles.

LET C be the Point propoſed in the Line AB, from which a Perpendicular is to be raiſed.

From the Point given C, draw at pleaſure any Semicircle EF; then from the Points EF, you muſt make a Section thus: Open your Compaſſes to the Diſtance EF, ſetting one Point in E, with the other deſcribe the Arch IG; then remove that Point of your Compaſs, and ſet it in F (keeping the Compaſſes at the ſame Diſtance) with the other Point thereof deſcribe the Arch IH, which will interſect or cut through the Arch IG; from which Interſection at I, a Line being drawn to the Point C, will be perpendicular, and cut the Line AB at right Angles.

This Prop. *may be performed, as in* Fig. II.

By opening your Compaſs to any convenient Diſtance, and ſetting one of the Points of the Compaſſes in the Point C, with the other Point make the prick A, then turning the Compaſſes, keeping one Point ſtill at C, with the other Point make the prick B in the Line given; then ſetting one Foot in A, and opening the Compaſſes to B, deſcribe the Arch HI; alſo remove the Compaſſes, and ſetting one Point in B, deſcribe another Arch HK, from which Interſection, draw the Line HC, which will cut the Line AB at right Angles.

PROP. II. Fig. III.

To raiſe a Perpendicular at the End of a Line.

LET the Line give be AB, and on the End B, it is requir'd to raiſe a Perpendicular.

Set one Foot of your Compaſſes above the Line AB at pleaſure, as ſuppoſe at D, and opening the other Point till it ſtay in B, with this Diſtance keeping the Point at D, deſcribe a Semicircle FBE; by the Points F and D, draw the ſtreight Line FDE; and where the ſtreight Line cuts the Circle as at E, from thence draw the Perpendicular EB.

To perform this Prop. *another way.* Fig. IV.

LET the Point B, at the End of the Line AB, be the Point from whence a Perpendicular is to be raiſed.

The Compaſſes being opened at Pleaſure, ſet one Foot in the Point B, and with the other Foot deſcribe the Arch of a Circle CDG, keeping the Compaſſes at the ſame Diſtance, ſetting one Foot in C, deſcribe the Arch BD, then ſetting one Foot in D, deſcribe the Arch FEB, and from the Point E, draw the Arch ID, and from the Interſection of this Arch, with the Arch FEB, draw the Perpendicular IB.

PROP. III. Fig. V.

To raiſe a Perpendicular at the End of a Line, having little or no Paper beyond that End of the Line, whereon you are to raiſe the Perpendicular.

LET AB, be the Line given, and B the End. Opening the Compaſſes at Pleaſure, ſetting one Foot in B, with the other Foot deſcribe the Arch CD; then remove one Foot of the Compaſs, and ſet it in C, and interſect the Arch CD at E; then lay a Ruler from C to E, and ſtrike a Line as EF; then ſet one Foot of your Compaſſes (being at their firſt Diſtance) in E, and with the other Foot make a Point or Prick in the Line EF at F, from whence draw the Perpendicular FB.

PROP. IV. Fig. VI.

Upon an Angle given to raiſe a Perpendicular.

LET ABC be the Angle given; ſetting one Foot of your Compaſſes in B, deſcribe the Arch AC; then opening the Compaſſes a little wider, and ſetting one Point in C, deſcribe the Arch DE; then removing the Point to A, interſect the Arch DE at F, from whence draw the Perpendicular FB.

PROP. V. Fig. VII.

To let fall a Perpendicular upon a Line given, and from a Point above the Line.

LET the Line given be AB, and the Point given be C; ſetting one Foot of the Compaſſes in C, deſcribe the Arch DE, and from D and E make the Interſection at F, from whence draw the Line CF.

PROP. VI. Fig. VIII.

By a Point given, to draw a Line parallel to a Right Line given.

LET A be the Point, by which we muſt draw a Line, which may be parallel to the Line BC. Draw at Pleaſure the Oblique (or Diagonal) Line AD; from the Point A draw the Arch DE, and from the Point D deſcribe the Arch AF, then ſetting one Point of the Compaſſes in the given Point A, extend the other Point to that Part of the given Line which is interſected by the Arch AF, with this Diſtance, ſetting one Point of the Compaſſes at the Interſection D, extend the other upon the Arch DE, and where that Point falls upon the Arch as at G, draw a Line from the Point A through the Point at G, and it will be parallel to the Line BC.

Another Way to draw a Parallel to the Line BC. Fig. IX.

THE Compaſſes being ſet to the Space you intend ſhall be between the two Lines (otherwiſe any Diſtance will ſerve) ſetting one Foot on the End of the Line B, with the other Foot deſcribe the Arch DE; this being done, keeping the Compaſſes at the ſame Diſtance, ſet one Point on the End of the Line C, and deſcribe the Arch FG, then draw the Line HI, juſt to touch the uppermoſt Part of theſe two Arches, and it will be parallel to the Line BC.

PROP. VII. Fig. X.

To cut a right Line AB *equally in the Middle.*

THE Compaſſes being opened to any Diſtance ſhorter than the whole Line, and longer than half the Line; as ſuppoſe them opened from A to C, ſetting one Point of the Compaſs in A, with the other deſcribe the Arch DE, keeping the Compaſſes at the ſame Diſtance, ſetting one Foot in B, with the other deſcribe the Arch FG, and from the Interſection (or cutting thro') of theſe two Arches, draw the Line HI, it will divide the Line AB equally in the Middle.

The Figures of the Propoſitions following, you will find in the next folded Page to this.

rop. I.

Fig: I.

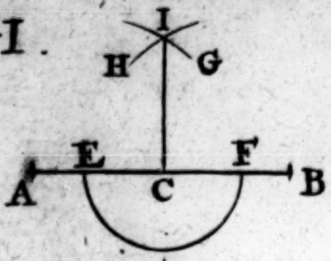


Fig: II.

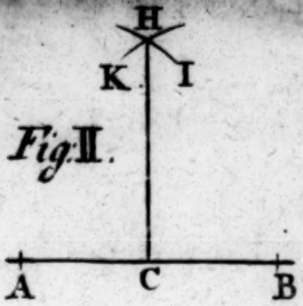


Fig: III.

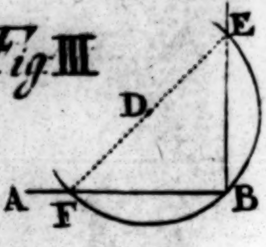


Fig: IIII.

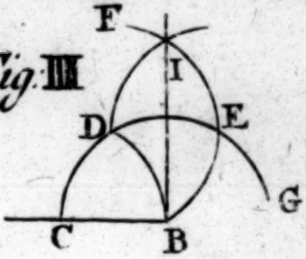


Fig: V.

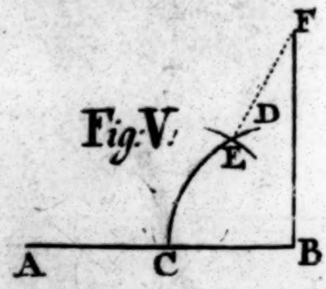


Fig: VI.

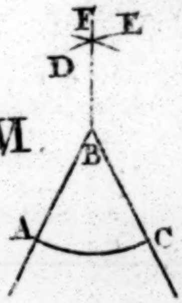


Fig: VII.

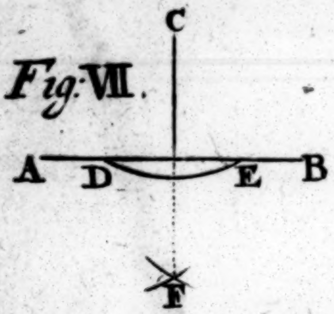


Fig: VIII.

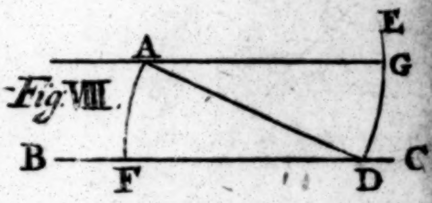


Fig: IX.

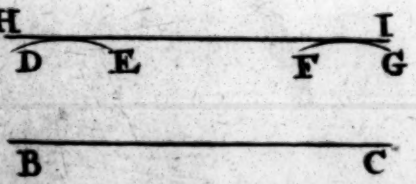
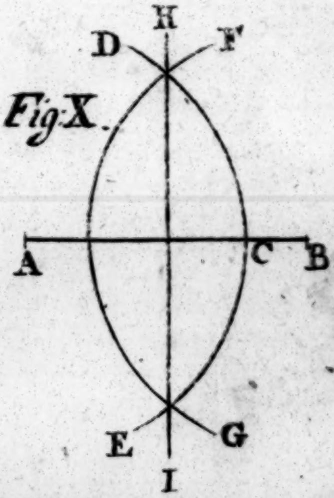


Fig: X.



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PROP. VIII. Fig. XI.

To cut a right lined Angle BAC, *into two equal Parts.*

OPening the Compaſſes at Pleaſure (*viz.* opening them to any Diſtance) ſetting one Point of them on the Angle A, with the other Point deſcribe the Arch DE, then from D and E, make the Interſection at G, through which Interſection, and the Angle A, draw the Line AG, which divides the Angles BAC into two equal Parts.

PROP. IX. Fig. XII.

To make a Triangle ABC, *of three right Lines* (viz. AB, BC, CA) *equal to three Lines given* D. E. F. *of which three Lines, any two being taken and added together, muſt be longer than the third Line remaining, otherwiſe you cannot make a Triangle of them.*

FIrſt draw the ſtrait Line GH, then upon that Line take GA, equal to the given Line D; alſo upon the ſame Line GH, ſet the Line E, from A to C; alſo ſet the given Line F, from C to D, then opening the Compaſſes to GA, with one Point in A, deſcribe the Circumference GBC; this being done, ſet one Point of the Compaſs at C, and the other being opened to D, deſcribe the Circumference DBI, then join the Triangle where theſe two Circles interſect, which is at B, and make the Triangle ABC, which is equal to the three Lines given D. E. F.

PROP. X. Fig. XIII.

At a Point A, *in a right Line given* AB, *to make a right lined Angle* A, *equal to a right lined Angle given* D.

SET one Point of the Compaſſes at D, and with the other deſcribe the Arch EF, then with the Compaſſes being at the ſame Diſtance, ſetting one Point at A, with the other Point deſcribe the Arch BC, then take the Chord Line of the Arch EF (*viz.* the ſtrait Line EF) between the Points of your Compaſſes, and ſet one Point at B, and where the other Point toucheth the Arch BC as at C, by that Point draw the Line AC, and the Angle A, will be equal to the Angle D.

PROP. XI. Fig. XIV.

To make a Parallelogram ABCD, *equal to a Triangle given* EFB, *in an Angle equal to a right lined Angle given* G.

THrough the Point E, draw the Line ED, parallel to the Line FB (by the 6 Prop.) then upon the Point B raiſe the Line BD, making the Angle at B, equal to the given Angle G, (as you were taught in the 10th Prop.) then Biſect (*viz.* divide in the Middle) the Baſe FB, as at A, and draw the Line AC parallel to BD, and the Parallelogram ABCD, will be equal to the Triangle given EFB, and like the Angle given G.

PROP. XII. Fig. XV.

Upon a right Line given A, *to make a Parallelogram* FL, *at a right lined Angle given* C, *equal to a Triangle given* B.

BY the foregoing Prop. make a Parallelogram FD, equal to the Triangle B, ſo that the Angle GFE may be equal to the Angle given C; continue the Line GF, 'till FH be produced equal to the given Line A; then by the Point H, draw the Line IL, parallel to EF, alſo continue the Line DE 'till it touch the Line HI, then draw the Diagonal Line IK, 'till it meet with the Line DG being continued, then through the Point K, draw the Line KL parallel to GH, then extend or continue the Line EF unto M, and the Line IH unto L, then ſhall FL be the Parallelogram required; for the Parallelogram FL is equal to the Parallelogram FD, and FD is equal to the Triangle given B, and the Angle MFH, is equal to GFE, and GFE is equal to the given Angle C.

PROP. XIII. Fig. XVI.

Parallelograms BCDA, GHFE, *ſtanding upon equal Baſes* BC, GH, *and betwixt the ſame Parallels* AF, BH, *are equal one to the other.*

DRaw BE and CF, becauſe BC is equal to GH, and GH equal to EF, therefore is BCFE a Parallelogram. Whence the Parallelogram BCDA is equal to BCFE, and that equal to GHFE, which was to be demonſtrated.

PROP. XIV. Fig. XVII.

Triangles BCA, BCD, *ſtanding upon the ſame Baſe* BC, *and between the ſame Parallels* BC, EF, *are equal one to the other.*

DRaw BE parallel to CA, and CF parallel to BD, then is the Triangle BCA, equal to half the Parallelogram BCAE, and alſo equal to half BDFC, and that equal to the Triangle BCD, which was to be demonſtrated.

PROP. XV. Fig. XVIII.

If a Parallelogram ABCD, *have the ſame Baſe* BC, *with the Triangle* BCE, *and be between the ſame Parallels* AE, BC, *then is the Parallelogram* ABCD, *double to the Triangle* BCE.

LET the Line AC be drawn. Then is the Triangle BCA equal to BCE: Therefore is the Parallelogram ABCD, equal to two ſuch Triangles as BCA, and likewiſe alſo equal to two ſuch Triangles as BCE, which was to be demonſtrated.

From hence may the Area (or Content) of any Triangle as BCE be found. For whereas the Area of the Parallelogram ABCD is produced by the Altitude drawn into the Baſe, therefore ſhall the Area of a Triangle be produced by Half of the Altitude drawn into its Baſe, or Half its Baſe drawn into its Altitude; as if ſo be, the Baſe BC be 8, and the Altitude 7, taking the Half of the Baſe 4, and multiplying it by the Altitude 7, it produceth 28, which is the Area or Content of the Triangle BCE; or otherwiſe, if you take the whole Baſe 8, and half the Altitude, 3 and an Half, and multiply them, they producc 28 (as before) for the Content.

PROP. XVI. Fig. XIX.

Upon a right Line given FG, *to make a Parallelogram* FL, *equal to a right lined Figure given* ABCD, *at a right lined Angle given* E.

REſolve the right lined Figure given into two Triangles BAD, BCD; then make a Parallelogram FH equal to BAD, ſo that the Angle F may be equal to the Angle E (as you were taught at the 12th Prop.) FI being produced to K, make the Parallelogram IL equal to the Triangle BCD. Then is the Parallelogram FL equal to FH more IL, and therefore equal to the Figure given ABCD, which was to be done.

SCHOL. Fig. XX.

HEnce is eaſily found the Exceſs HE, whereby any right lined Figure A, exceeds a leſs right lined Figure B; namely, If to ſome right Line CD both be applied, and both the Trapezia's, each of them being divided into two Triangles, and working as before is taught, you will find the Parallelogram DF equal to the Trapezia A, and the Parallelogram DH equal to the Trapezia B, ſo that the Figure A exceeds the Figure B by ſo much as the Parallelogram GEFH contains.

PROP.

PROP. XVII. Fig. XXI.

In right Angled Triangles BAC, *the Square* BE, *which is made of the Side* BC *that ſubtends the right Angle* BAC, *is equal to both the Squares* BG *and* CH, *which are made of the Sides* AB, AC, *containing the right Angle.*

JOin AE and AD, and draw AM parallel to CE, becauſe the Angle DBC is equal to FBA, add the Angle ABC common to them both, then is the Angle ABD equal to FBC. Moreover AB is equal to FB, and BD equal to BC; therefore is the Triangle ABD equal to FBC. But the Parallelogram BM, is equal to two ſuch Triangles as ABD, and the Parallelogram or Square BG, is equal to two ſuch Triangles as FBC (for GAC is one right Line by the Hypotheſis) therefore is the Parallelogram BM equal to the Quadrate BG. By the ſame Way of Argument, is the Parallelogram CM equal to the Quadrate CH; therefore is the whole Parallelogram (or Square) BDEC equal to the two Quadrates BAGF and ACIH, which was to be done.

PROP. XVIII. Fig. XXII.

There are Three Quadrates or Squares given, whereof the Sides are AB, BC, CE, *and it is required to make one Square, whoſe Area or Content ſhall be equal to the Area of thoſe Three Squares.*

MAke the right Angle FBZ, having the Sides infinite (the Meaning of infinite is to draw the Sides long enough, and of what Length there is no Determination) and on theſe two Sides tranſſer AB and BC; that is to ſay, Take the given Line AB between your Compaſſes, and place it from the Angle B to A; alſo take the Line given BC, and place it from the Angle B to C; join AC (*viz.* draw the Line AC) then is a Square, whoſe Side is AC, equal to two Squares made of the two Lines AB and BC, then take the Line AC and place it from B to X; alſo take the third given Line or Side CE, and place it from B to E, then draw the Line EX, then a Square being made whoſe Side is EX, is equal to three Squares, being made of the three Lines or Sides given, AB, BC, CE, which was to be done. The Truth whereof is manifeſted by Arithmetick; for let the Line AB be 8 Feet in Length, then the Line BC will be 5 Feet, and the Line CE will be 4 Feet, and the Line EX will be 10 Feet and 6 Inches: Now the Square of 8 is 64, and the Square of 5 is 25, and the Square of 4 is 16, which three Squares being added together, produce 105, for the Area of the three given Squares; therefore the Square of the Line EX being 10 Feet and 6 Inches, is 105 Feet, and equal to the three Squares given.

PROP. XIX. Fig. XXIII.

Two unequal right Lines being given AB, BC; *to make a Square equal to the Difference of the two Squares of the given Lines* AB, BC.

FRom the Centre B, with the Diſtance BA deſcribe a Semicircle, and from the Point C erect a Perpendicular CE, meeting with the Circumference in E, and draw BE. Then is the Square of BE (or BA) equal to the Square of BC and CE. Therefore when the Square of BC is taken out of the Square of BA, the remaining Part of the Square of BA will be equal to the Square of CE, which was to be done.

PROP. XX. Fig. XXIV.

Any two Sides of a right Angled Triangle ABC, *being known, to find out the third.*

LET the Sides AB, AC, encompaſſing the right Angle, be the one 6 Feet, the other 8 Feet: Therefore, whereas the Square of AB is 36, and the Square of AC is 64, which being added, make 100, therefore (as you were taught at the 17th Prop.) the other Side ſought for, muſt be equal in Power (*viz.* being ſquared) to the two given Sides being ſquared, which contain 100, whoſe Square Root is 10, the Length of the Side ſought BC, which was to be done.

PROP. XXI. Fig. I.

To deſcribe a Circumference that ſhall touch any three Points given, provided they are not in a right Line; ſuppoſe the Points given, to be A. B. C. *(The Figures of this, and the following Propoſitions, you will find in the next folded Page.)*

TAke the Diſtance between A and B with your Compaſſes, and ſetting one Point of the Compaſs on the Point by A, deſcribe the Arch DE; then with the ſame Diſtance ſetting one Point of the Compaſs on the Point by B, deſcribe another Arch, which will cut the former Arch in the Points by 1; then laying a Ruler to the Points by 1, where the Arches interſect, draw a ſtrait Line FG. This being done, take with your Compaſſes the Diſtance between the other Point C and B, and with this Diſtance, ſetting one Point in B, deſcribe the Arch HI; then with the Compaſſes at the ſame Diſtance, ſetting one Point on the Prick by C, deſcribe another Arch, cutting the former in the Point, and by 2, thro' which Points draw another ſtrait Line, 'till it cut through the firſt ſtrait Line, as at G; I ſay G is the Centre, from whence the Circumference A. B. C. is deſcribed, which toucheth the three given Points.

Prop. VIII

Fig: XI.

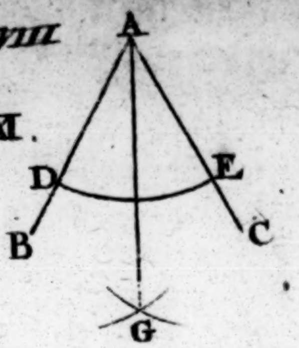


Fig: XII

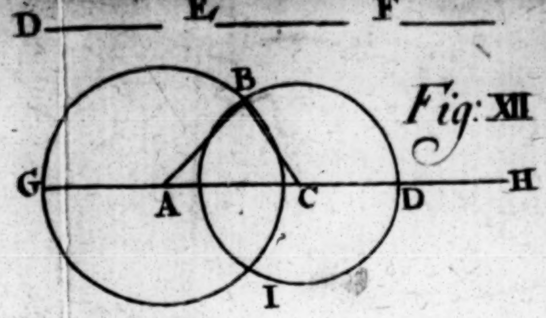


Fig: XXI.

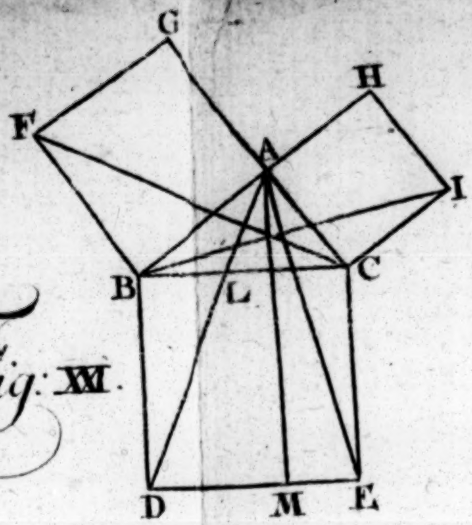


Fig: XXII.

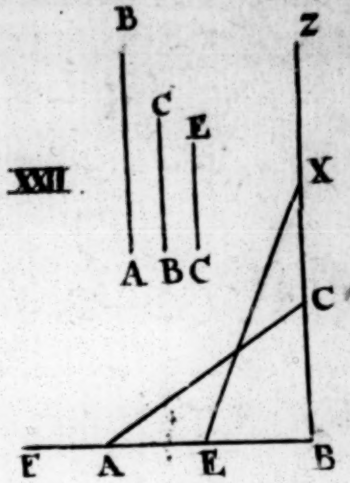


Fig: XIII.

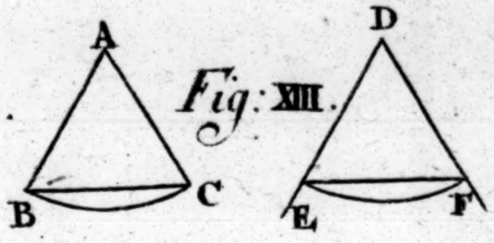


Fig: XIV

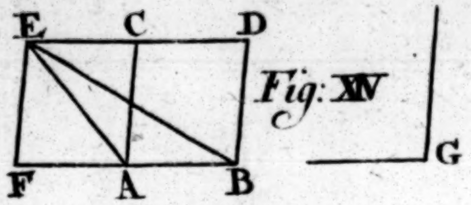


Fig: XV.

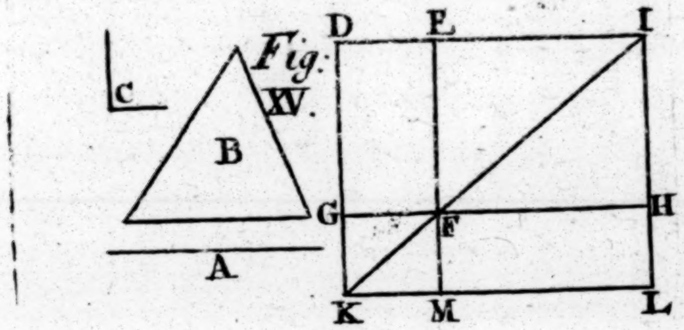


Fig: XVI.

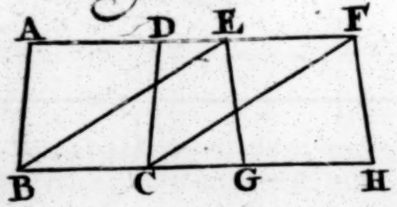


Fig: XXIII.

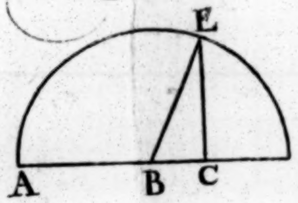


Fig: XXIIII.

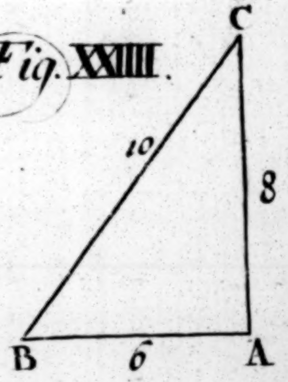
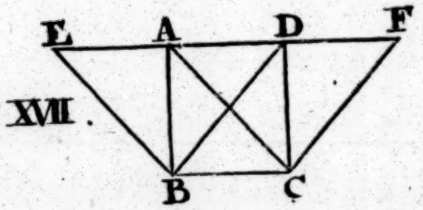


Fig: XVII.



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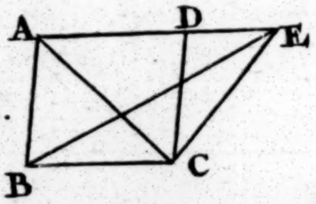

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Fig: XIX.

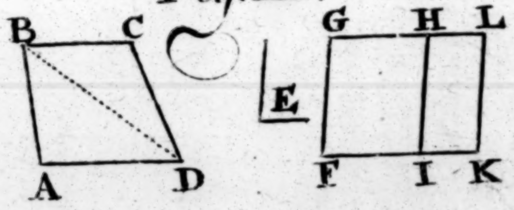
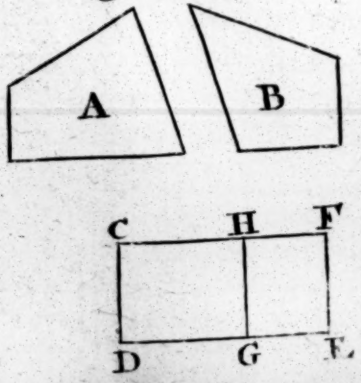


Fig: XX



PROP. XXII. Fig. II.

To deſcribe an Oval upon a Length given AB.

DIvide the given Line into three equal Parts (as the next Propoſition following will teach) ACDB, ſetting one Point of the Compaſſes at the Point by C, with the Diſtance CA, deſcribe the Circle AEF, then with the ſame Diſtance ſetting one Point by D, deſcribe the Circle BEF, then draw four ſtrait Lines thro' the Centres C and D, and the Interſection of the two Circles E and F; then ſetting one Point of the Compaſſes in E, and extending the other Point to I, deſcribe the Arch IH, then with the Compaſſes at the ſame Diſtance, ſetting one Point on the Interſection by F, deſcribe the Arch OP, which concludes the Oval. *Note*, That IAO and HBP, are vulgarly called Hanſes, and IH and OP, are called Schemes.

PROP. XXIII. Fig. III.

To divide a ſtrait Line given AB, *into three equal Parts.*

FRom the End A, draw at Pleaſure the Line AC, making what Angle you will, then from the other End of the Line B, draw the Line BD, parallel to the Line AC; then opening your Compaſſes at Pleaſure, ſetting one Point in A, turn them three Times over the Line AC, which will make three Diviſions, *viz.* EFG, then with the Compaſſes continuing at the ſame Diſtance, ſetting one Foot in B, make three Diviſions on the

Line BD, *viz.* HIK, then draw with a Ruler, and the Point of the Compaſs, a ſtrait Line from A to K, another from E to I, alſo another from F to H, and another from G to B; and they being drawn, the given Line AB is divided into three equal Parts, which was to be done. You may, if you pleaſe, divide the ſame Line, or any other, into what Number of equal Parts you pleaſe, by dividing the two parallel Lines AC and BD, into ſo many equal Parts as you would have the given Line divided into.

PROP. XXIV. Fig. IV.

To deſcribe an Oval equal in Length to the first Oval not riſing ſo high.

DIvide the given Line AO into four equal Parts (by the foregoing Prop.) in BCD, then taking one of thoſe Parts between the Compaſſes, upon the Centres BCD, deſcribe three Circles, and thoſe 2 Parts of the middlemoſt Circle, that is without the 2 other Circles, divide in the Middle at E and F, then from E to the Centre D, draw a ſtrait Line, and contine it to the Circumference at 4, alſo draw another ſtrait Line from E, thro' the Centre by B to the Circumference, which will cut it at 3; likewiſe from F thro' the ſame Centres draw right Lines, which will cut the two Circumferences, the one in 2, the other in 1. Then from the Centre F with the Radius (or Diſtance) F, 1, deſcribe the Arch 1, 2; alſo from the Centre E with the ſame Radius, deſcribe the Arch 3, 4, which concludes the Oval.

PROP.

PROP. XXV. Fig. V.

Another Way to deſcribe Ovals.

UPon the Line given, deſcribe two equilateral Triangles, join them together with one common Baſe, ſo that they make a Rhombus; then continue (or draw) the Line AC to 3, ſo that C 3 may be 6, ſuch Parts whereof AC is 5, *viz.* It muſt be the Length of AC, and one 5th Part more of it. Alſo draw the Line BD to 2, that it may be the ſame Length with A 3; alſo draw BC to 1, and AD to 4, being all of one Length: Then from the Centre A, with the Radius A 3, deſcribe the Arch 3, 4; alſo from the Centre B, with the ſame Radius, deſcribe the Arch 1, 2; then from the Centre C, with the Radius C 1, deſcribe the Arch 1, 3; likewiſe from the Centre D, with the ſame Radius, deſcribe the Arch 2, 4, which will incloſe the Oval. From theſe 4 Centres you may deſcribe Ovals, greater or leſſer as you pleaſe.

PROP. XXVI. Fig. VI.

To deſcribe an Oval according to any Length and Breadth given.

LET the Length given be AB, and the Breadth CD.

Apply the 2 given Lines together, ſo that they may cut each other into 2 equal Parts, and at right Angles in the Point E; then take half the Line AB between your Compaſſes, and ſetting one Point of the Compaſſes in C, extend the other till it touch the Line AB, in K and L, which 2 Points are called the burning Points, or Focus's. In which Points, drive 2 Nails if you deſcribe it upon Boards, but upon Paper, as here, 2 Pins will do; the Pins being ſtuck firm in the Points K and L, ſtick alſo another Pin in the Point C; then take a Thread and encompaſs theſe 3 Pins in Form of a Triangle, pulling the Thread tight, tie the 2 Ends of the Thread together by a Knot at C; then taking out the Pin at C, take a Pencil, holding it cloſe to the Inſide of the Thread, and carrying the Pencil round upon the Paper, about the Pins, with the Thread always ſtrait, the Ellipſis or Oval ACBD ſhall be thereby deſcribed.

PROP. XXVII. Fig. VII.

To find the Centre and the two Diameters of an Oval.

LEt SETD be the Oval whereof the Centre and the Diameters are to be found.

Within the Oval, draw at Diſcretion the Parallel Lines EF, GH; cut theſe Lines into 2 equally in I and K, draw the Line IK, cut it into 2 equally in L, which is the middle Centre of the Oval; upon this Centre L, deſcribe at Pleaſure the Circle MNO, cutting the Oval in P and Q, from which Sections, draw the right Line PQ, cut it in the Middle in R, from which, through the Centre L, draw the greater Diameter ST, and from the Centre L, draw the leſſer Diameter ELD parallel to the Line PQ, which was to be done.

PROP.

PROP. XXVIII. Fig. VIII.

To deſcribe an Oval with a pair of Compaſſes, to any Length and Breadth given.

I Shall only deſcribe a Semi-oval, and according to the ſame Rules, if you will, you may deſcribe the whole Oval.

Let the Length given be A B, and one half of the Breadth C D; divide A B into ſeven equal Parts; then upon one ſeventh Part from A, as at E, raiſe a Perpendicular from the Line A B (*viz.* E G.) Alſo at one ſeventh Part from B, as at F, raiſe another Perpendicular F H; then divide the half Breadth given C D, into fifteen equal Parts, and take eleven of thoſe Parts and ſet upon the Perpendicular from E to G, and likewiſe from F to H; then taking the Space between A and G, ſetting one Point of the Compaſſes in A, deſcribe the Arch G i; keeping the Compaſſes at the ſame Diſtance, ſet one Point in G, and deſcribe another Arch, which will cut the former in the Point by i; from which Point, with the Radius A G, deſcribe the Semi-hanſe A G. This being done, take between your Compaſſes the Space B H, and ſetting one Point in B, deſcribe the Arch I i; then remove your Compaſſes to H, and interſect that Arch in the Point by i; then ſetting your Compaſſes on the Point i, with the ſame Diſtance, deſcribe a Part of the Oval B H, which part, as alſo the other part A G, are vulgarly called Semi-hanſes, becauſe it is but a Semi-oval (Semi ſignifies half) but if it had been an whole Oval, then the Semi-hanſe above the Line A, and another Semi-hanſe below the

Line A, being joined, is called an Hanſe, from the *Latin* Word *Hanus*, ſignifying a great-bellied thing. The other part to be deſcribed, from G to H, is called the Scheme, which to deſcribe, continue or draw longer the half Breadth D C, and in that Line find a Centre, whereon ſetting one Point of the Compaſſes, the other Point may touch the three Points G, D, H, as on the Centre I, whereby deſcribe the Scheme G D H, which was to be done.

CHAP. III.

A Digreſſion concerning Ellipſis Arches.

AND ſince Ellipſis or Semi-oval Arches, being neatly wrought in Brick, ſhew very pleaſant; and are ſometimes uſed over Gate-ways, and ſometimes over Kitchen Chimneys, inſtead of Mantle-trees; I think fit to write ſomething concerning them, relating to Bricklayers making the Moulds, and dividing the Courſes.

The Ellipſis you may deſcribe to what Length and Heighth you pleaſe, either by the laſt Propoſition, or by the 26th.

We will ſuppoſe an Ellipſis Arch to be made over a Chimney, whoſe Diameter between the Jaums is 8 Feet, and the under Side of the Arch, at the Key, to riſe in Height 18 Inches, from the Level of the Place whence you begin to ſpring the Arch. The Height or Depth of the Arch, we will ſuppoſe to be made of the Length of two Bricks, which when they are cut to the Sweep of the Arch,

will

Prop. XXI.

Fig: I.

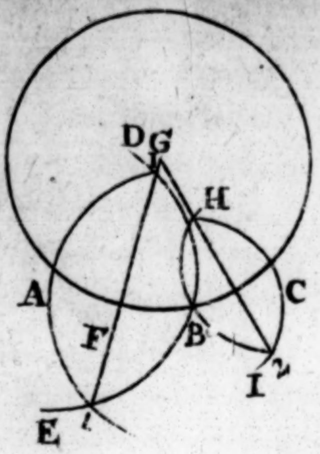


Fig: II.

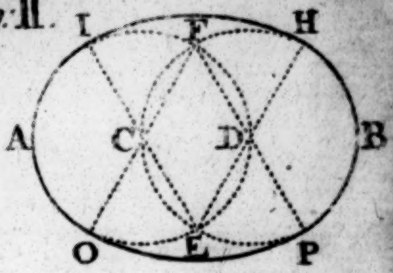


Fig: IIII.

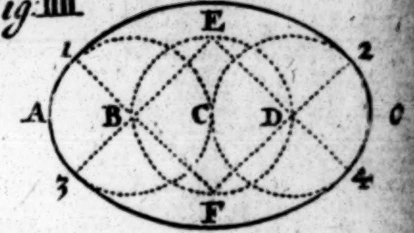
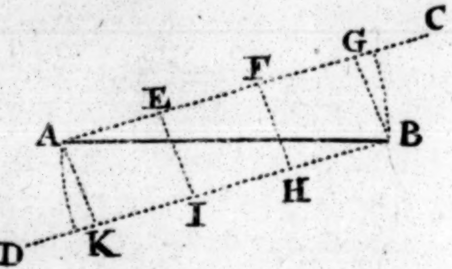


Fig: III.



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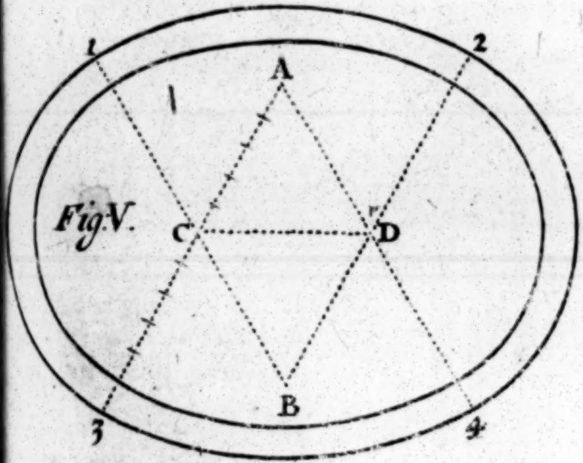
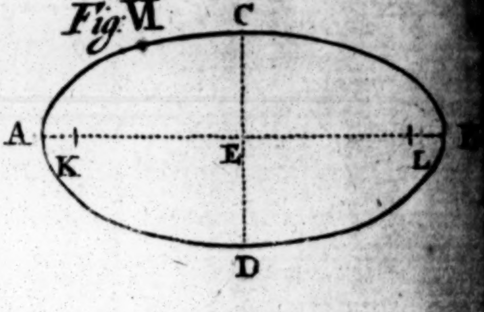


Fig: VI.



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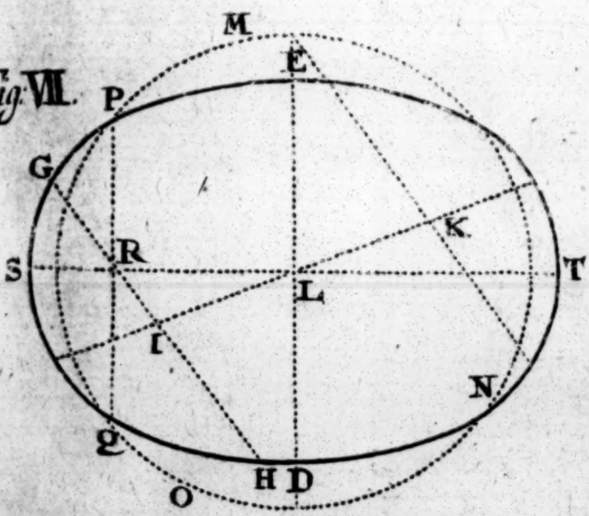
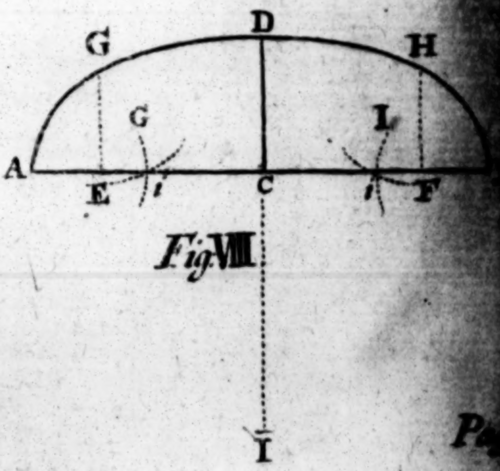


Fig: VIII.



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will not contain above 14 Inches; and perhaps you muſt cement Pieces to many of the Courſes in the Hanſe, to make them long enough to contain or hold 14 Inches, eſpecially if you intend to make the Courſes of the Hanſe, and the Courſes of the Scheme to ſeem alike in Greatneſs on the under Side of the Arch. For if you make the Hanſe to come to a true Sommering for the Scheme, by that time that you have ended the Hanſe, and are ready to ſet the firſt Courſes of the Scheme, the Mould, and ſo likewiſe each Courſe in the Hanſe, will be much leſs at the lower Part, or under Side of the Arch, than the Mould or Courſes of the Scheme; as you may perceive by the Hanſe B K, in the IX *Fig.* which Way of working theſe kind of Arches is ſtronger, than to make the Courſes ſeem alike in Bigneſs in Hanſe and Scheme, altho' it be not ſo pleaſing to the Eye. In the IX *Fig.* I will ſhew how to make one Half of the Arch this Way, and in the other Half ſhew how to make the Courſes in Hanſe and Scheme of a Bigneſs.

Firſt, Deſcribe the under Side of the Arch (*viz.* the Ellipſis A D B, whoſe Diameter A B is 8 Feet, and the Height C D 18 Inches) upon ſome ſmooth Floor, or ſtrait plaiſtered Wall, or ſuch like; then continue (*viz.* draw longer) both the Lines A B, C D, cutting each other at right Angles; then from A to E; alſo from B to F; likewiſe from D to G, ſet 14 Inches, the intended Height of your Arch. Then deſcribe another Ellipſis to that Length and Height, after this manner: Lay a ſtrait Ruler on the Centre by I, and on the joining of the Hanſe and the Scheme togethet, as at K, and draw the Line K L; then ſet one Point of your

Compaſſes in the Centre of the Hanſe at M, and open the other Point of the Compaſſes to F, and deſcribe the upper Hanſe F L; likewiſe ſetting one Point of the Compaſſes in the Centre by I, with the other extended to G, deſcribe the Scheme G L; (altho' I ſpeak here of Compaſſes, yet when you deſcribe an Arch to its full Bigneſs, you muſt make uſe of Centre Lines, or Rules; the laſt are beſt, becauſe Lines are ſubject to ſtretch) then taking between your Compaſſes the Thickneſs of a Brick, abating ſome ſmall Matter, which will be rubb'd off from both Beds of the Brick, with the Compaſſes, at this Diſtance, divide the upper Hanſe from L to F into equal Parts, and if they happen not to divide it into equal Parts, then open them a ſmall matter wider, or ſhut them a ſmall matter cloſer, till it doth divide it into equal Parts, and look how many equal Parts you divide the upper Hanſe into, ſo many equal Parts you muſt divide the lower Hanſe from K to B into likewiſe (or you may divide the upper Hanſe from the Centre O, making a right Angle from each Sommering Line to the Ellipſis, as is ſhewn in deſcribing the ſtreight Arches following; and from the Centre O, and the Diviſions in the upper Hanſe being thus divided, you may draw the ſtrait Lines to the lower Hanſe, and not divide it with the Compaſſes) thro' each of which Diviſions, with a Rule and Pencil, draw ſtrait Lines; then get a Piece of thin Wainſcot, and make it to fit between two of theſe Lines, allowing what thickneſs for Mortar you intend, this will be the Sommering Mould for the Hanſe; then divide the upper Scheme likewiſe, with the Compaſſes, at the ſame Diſtance, into equal Parts, and laying a Ruler on the Centre I, from each Diviſion in the Scheme G L, draw ſtrait Lines to the lower Scheme D K, then make another Sommering Mould to fit between two of theſe Lines, abating ſo much as you intend the Thickneſs of your Joints of Mortar to be, which, if you ſet very cloſe Mortars, the Breadth of the Line will be enough to allow: then laying the inner Edge of a Bevil ſtrait on the Line K L, bring the Tongue to touch the under Side of the firſt Courſe of the Scheme; then take up the Bevil, and ſet that Bevil Line upon the Sommering Mould of the Scheme, which Bevil Line ſerves for each Courſe in the Scheme: but you muſt take the Bevil of each Courſe in the Hanſe, and ſet them upon your Sommering Mould, and number them with 1, 2, 3, 4, *&c.* becauſe each Courſe varies.

Thus having made your Sommering Moulds, in the next place you muſt make the Moulds for the Length of your Stretchers, and for the Breadth of the Headers and the Cloſiers. A Piece of Wainſcot 7 Inches long, and 3 Inches and an half broad, will ſerve for the Length of the Stretchers, and the Breadth of the Headers; the Cloſiers will be 1 Inch ¾ broad.

So the Cloſier will be half the Breadth of the Header, and the Header half the Length of the Stretcher, which will look well.

It remains now to ſpeak ſomething to the other part of the Arch, to wit, A D, whoſe Courſes both in Hanſe and Scheme run alike upon the Ellipſis Lines, and ſeem of one Bigneſs (altho' perhaps there may be ſome ſmall matter of Difference, by reaſon I have not divided the Courſes in this Figure from a right Angle, but every Courſe from the Angle, which it makes with the Ellipſis, which I choſe rather to do, that ſo the Bevil of one Courſe might not ſeem to run more upon the Ellipſis than the Bevil of another, and the Difference of the Thickneſſes being ſo inconſiderate, is not diſcerned.)

Having deſcribed both the Ellipſis Lines A D, E G, divide each of them into a like Number of equal Parts, always remembring to make each Diviſion on the upper Ellipſis Line no greater than the Thickneſs of the Brick will contain, when it is wrought; then through each Diviſion, in both the Ellipſes, draw ſtrait Lines, continuing them 4 or 5 Inches above the upper Ellipſis Line, and as much below the lower Ellipſis Line: Then having provided ſome thin Sheets of fine Paſtboard about 20 Inches ſquare, cutting one Edge ſtrait, take one Sheet, and lay the ſtrait Edge even upon the Line A E, ſo that it may cover both the Ellipſis Lines, and being cut to Advantage, it may cover 8 Courſes (or 9 of the ſtrait Lines.) Having laid it thus upon the Figure of the Arch, ſtick a Pin or two through it, to keep it in its Place; then lay a Ruler upon the Paſtboard to the 7th, 8th, or 9th ſtrait Line of the Arch, according as the Paſtboard be in Bigneſs to cover them, and take a ſharp Penknife, laying the Ruler upon the Paſtboard true to the ſtrait Line (whoſe Ends being continued longer than the Arch is deep, as I directed before, will be ſeen beyond the Paſtboard) and cut the Paſtboard true to the Line; then take another Sheet and join to it, and cut it as you did the firſt, and ſo continue till you have covered the Arch from A E juſt to the Line D G, ſticking Pins in each Sheet to keep them in the Places where you lay them: Then deſcribe both the Ellipſis Lines upon the Paſtboard, from the ſame Centers and Radii that you deſcribed the Ellipſes under the Paſtboard, and either divide the Ellipſis Lines with the Compaſſes on the Paſtboard, or elſe draw Lines upon the Paſtboard from or by the ſtrait Lines underneath whoſe Ends you ſee; but the ſurer Way is to divide the Ellipſes on the Paſtboard, and draw Lines through thoſe Diviſions as you did beneath the Paſtboard; then ſet 7 Inches, being the Length of each Stretcher, from A towards E, and from D towards G, and deſcribe, from the former Centres, the Ellipſis *o o* thro' each other Courſe on the Paſtboard, as you may ſee in the Fig. alſo ſet 3 Inches and an Half, being the Breadth of the Header from A towards E, and likewiſe from D towards G; alſo ſet the ſame 3 Inches and an Half from E towards A, and from G towards D, and deſcribe theſe two Ellipſis Lines from the ſame Centres thro' each Courſe which the Ellipſis Line of the Stretchers miſs'd; likewiſe draw in the ſame Courſes, two other Ellipſis Lines one Inch and $\frac{3}{4}$ from each of thoſe two Lines you drew laſt; which is the Breadth of the Cloſiers; thus one Courſe of the Arch will be divided into two Stretchers, and the next to it into three Headers, and two Cloſiers through the whole Arch. This being done, cut the Paſtboard according to the Lines into ſeveral Courſes, and each other Courſe into two Stretchers, and the Heading Courſe into three Headers, and two Cloſiers, exactly according to the Sweep of the black Lead Lines, and mark each Courſe with Figures, marking the firſt Courſe of the Hanſe with 1, the next with 2, the third with 3, and ſo continue till you have marked all the Courſes to the Key or Middle, for every Courſe differs; you were beſt to mark the lower Cloſier in each Courſe with a Cypher on the left Hand of its own Number, that you may know it readily from the upper Cloſier, and make no Miſtakes when you come to ſet them; alſo the middle Headers in each Courſe ſhould be marked beſides it own Number, the Thickneſs of the upper Header being eaſily diſcern'd from the lower Header needs no marking beſides its own Number: The croſs Joints, and likewiſe the under ſide and upper ſide of each Courſe muſt be cut circular, as the Paſtboards, which are your Moulds, direct you.

If you will add a Keyſtone and Chaptrels to the Arch, as in the Figure; let the Breadth of the upper part of the Keyſtone be the Height of the Arch, *viz.* 14 Inches, and Sommer, from the Center at I, then make your Chaptrels the ſame Thickneſs that your lower part of the Keyſtone is, and let the Keyſtone break without the Arch, ſo much as you project or ſail over the Jaums with the Chaptrels.

Other kind of Circular Arches, as half Rounds and Schemes, being deſcribed from one Centre, are ſo plain and eaſy, that I need ſay nothing concerning them: But ſince ſtrait Arches are much uſed, and many Workmen know not the true Way of deſcribing them, I ſhall write ſomething briefly concerning them.

Strait Arches are uſed generally over Windows and Doors, and according to the Breadth of the

Piers between the Windows, ſo ought the Skew-back, or Sommering of the Arch to be; for if the Piers be of a good Breath, as 3 or 4 Bricks in Length, then the ſtrait Arch may be deſcrib'd (as it's vulgarly called) from the *Oxi*, which being but part of a Word is taken from the Word *Oxigonium*, ſignifying an equilateral Triangle with three ſharp Angles; but if the Piers are ſmall, as ſometimes they are but the Length of two Bricks, and ſometimes but one Brick and an Half, then the Breadth of the Window, or more, may be ſet down upon the middle Line for the Centre, which will give a leſs Skew-back or Sommering than the Centre from an *Oxi*. I will ſhew how to deſcribe them both Ways, and firſt from the *Oxi*.

Suppoſe a ſtrait Arch 1 Brick and an Half in Height, to be made over a Window, 4 Feet in Width. [See *Fig.* X.] wherein one Half of the Arch is deſcribed from the Oxi, and the other Half from the Width of the Window. Let the Width of the Window be A B; taking the Width between the Compaſſes, from A and B as two Centres, deſcribe the two Arches, interſecting each other at P, (though I ſpeak here of Compaſſes, yet when you deſcribe the Arch to its full Bigneſs, you muſt uſe a Ruler or a Line, ſcarce any Compaſſes being to be got large enough) then draw another Line above the Line A B, as the Line C D, being parallel to it, at ſuch a Height as you intend your Arch to be, as in this *Fig.* at 12 Inches; but moſt commonly theſe ſort of Arches are but 11 Inches in the Height, or thereabouts, which anſwers to 4 Courſes of Bricks, but you may make them more or leſs in Height according as Occaſion requires;

then laying a Ruler on the Centre P, and on the End of the Line A, draw the Line A C, which is vulgarly called the Skew-back, for the Arch.

The next thing to be done, is to divide thoſe two Lines A B and C D into ſo many Courſes as the Arch will contain, the Thickneſs of a Brick being one of them, which ſome do by dividing the upper Line into ſo many equal Parts, and from thoſe Parts, and from the Centre P, draw the Sommering Lines or Courſes; others divide both the upper and lower Line into ſo many equal Parts, and make no uſe of a Centre, but draw the Courſes by a Ruler, being laid from the Diviſions on the upper Line, to the Diviſions on the lower Line, both which Ways are falſe and erroneous; [but this by Way of Caution.]

Having drawn the Skew-back A C, take between your Compaſſes the Thickneſs that a Brick will contain, which I ſuppoſe to be two Inches when it is rubb'd, and ſetting one Point of the Compaſſes on the Line C D, ſo that when you turn the other Point about, it may juſt touch the Line A C in one place, and there make a Prick in the Line C D, but do not draw the Sommering Lines until you have gone over half the Arch, to ſee how you come to the Key or Middle; and if you happen to come juſt to the middle Line, or want an Inch of it, then you may draw the Lines, but if not, then you muſt open or ſhut the Compaſſes a little till you do.

Then keeping one End of the Rule cloſe to the Centre at P, (the ſureſt Way is to ſtrike a ſmall Nail in the Centre P, and keep the Rule cloſe to the Nail) lay the other End of the Rule cloſe to

th

the Prick that you made on the Line C D, keeping the Compaſſes at the ſame Width (*viz.* 2 Inches) ſet one Point of the Compaſſes on the Line C D as before, ſo that the other Point being turned about may juſt paſs by the Rule, and, as it were, touch it in one Place (you muſt remove the Point of the Compaſſes upon the Line C D, farther or nearer to the Rule, until it juſt touch the Rule in one Place) and ſo continue with the Rule and Compaſſes until you come to the middle Line, and if it happen that your laſt Space want an Inch of the Middle, then the Middle of the Key-courſe will be the Middle of the Arch, and the Number of the Courſes in the whole Arch will be odd; but if the laſt Space happen to fall juſt upon the middle Line E F, as it doth in the Fig. then the Joint is the Middle of the Arch, (but if it ſhould happen neither to come even to the Line, nor want an Inch of it, then you muſt open or ſhut the Compaſſes a ſmall matter, and begin again till it doth come right) and the Number of the Courſes in the whole Arch is an even Number.

Note, When the Number of all the Courſes in the Arch is an even Number, then you muſt begin the two Sides contrary, *viz.* A Header to be the lower Brick of the firſt Courſe on one Side (or half) of the Arch, and a Stretcher the lower Brick of the firſt Courſe on the other Side (or Half) of the Arch; And contrariwiſe, if it happen that the Number of the Courſes be an odd Number, as 25 or 27, or ſuch like, then the firſt Courſes of each Half of the Areh muſt be alike, that is, either both Headers or both Stretchers, at the Bottom.

Thus having deſcribed the Arch, the next thing to be done is to make the Sommering Mould, which to do, get a Piece of thin Wainſcot (being ſtrait on one Edge, and having one Side plained ſmooth to ſet the Bevil Strokes upon) about 14 Inches long, and any Breadth above 2 Inches; then laying your Ruler, one End at the Centre P, and the other End even in the Skew-back Line, clap the ſtrait Edge of the Wainſcot cloſe to the Rule, ſo that the lower End of the Wainſcot may lie a little below the Line A B; then take away the Centre Rule, but ſtir not the Wainſcot, and laying a Ruler upon the Wainſcot, juſt over the Line C D, ſtrike a Line upon the Wainſcot; then ſet one Point of the Compaſſes, being at the Width of a Courſe (*viz.* 2 Inches) upon that Line, ſo that the other Point being turned about, may juſt touch the ſtrait Edge of the Wainſcot (as you did before in dividing the Courſes) then make a Prick on the Line on the Wainſcot, and laying your Centre Rule upon it, and on the Centre P, draw a Line upon the Wainſcot by the Ruler with a Pencil, or the Point of a Compaſs, and cut the Wainſcot to that Line, and make it ſtrait by ſhooting it with a Plane; then your Wainſcot will ſit exactly between any two Lines of the Arch. You may let it want the Thickneſs of one of the Lines, or ſome ſmall Matter more, which is enough for the Thickneſs of a Mortar. The Length of your Stretcher in this Arch may be 8 Inches and ¼, and the Header 3 Inches and ¾, but if your Arch be but 11 Inches in Height, then make your Stretcher 7 Inches and an Half long, and the Header 3 Inches ½. One Piece of Wainſcot will ſerve both for the Length of the Scretcher, and the Length of the Header, making it like a long Square, or Oblong, whoſe Sides are 8 Inches ¼, and 3 Inches and ¾.

Then

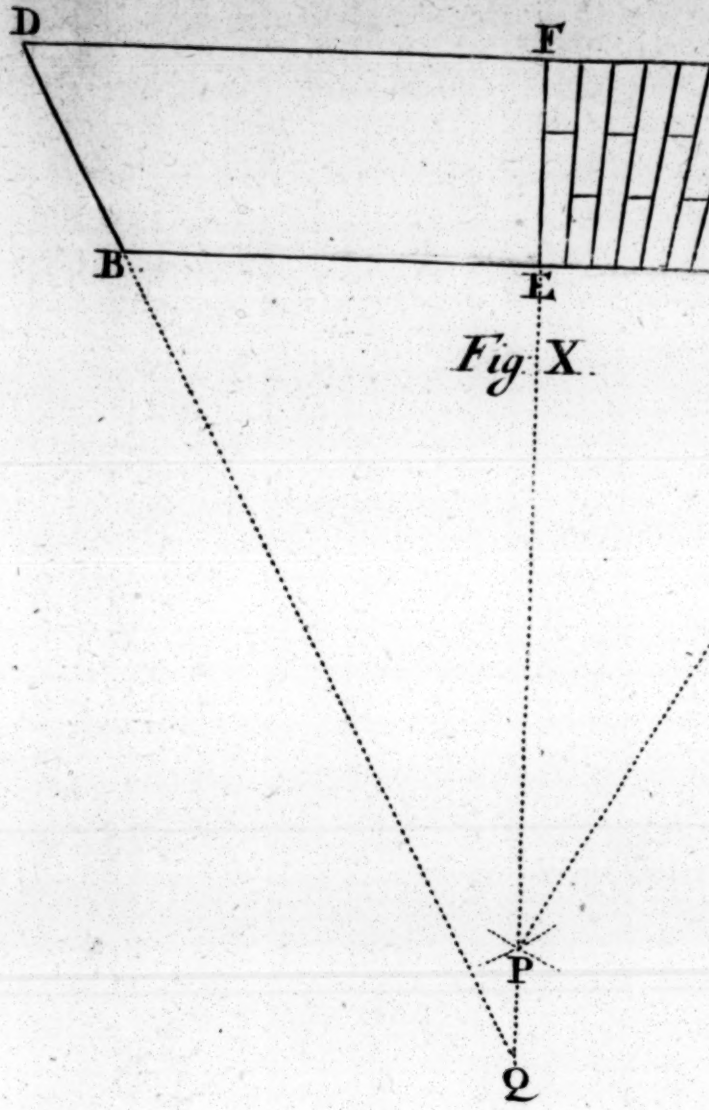


Fig X.

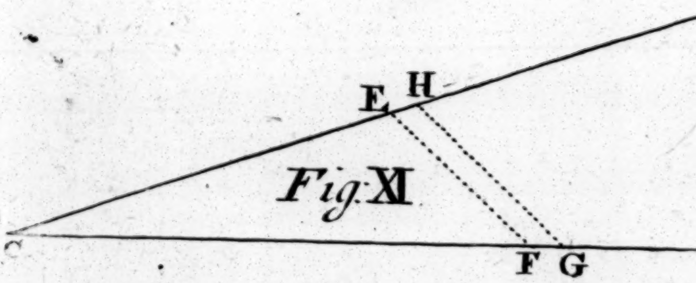
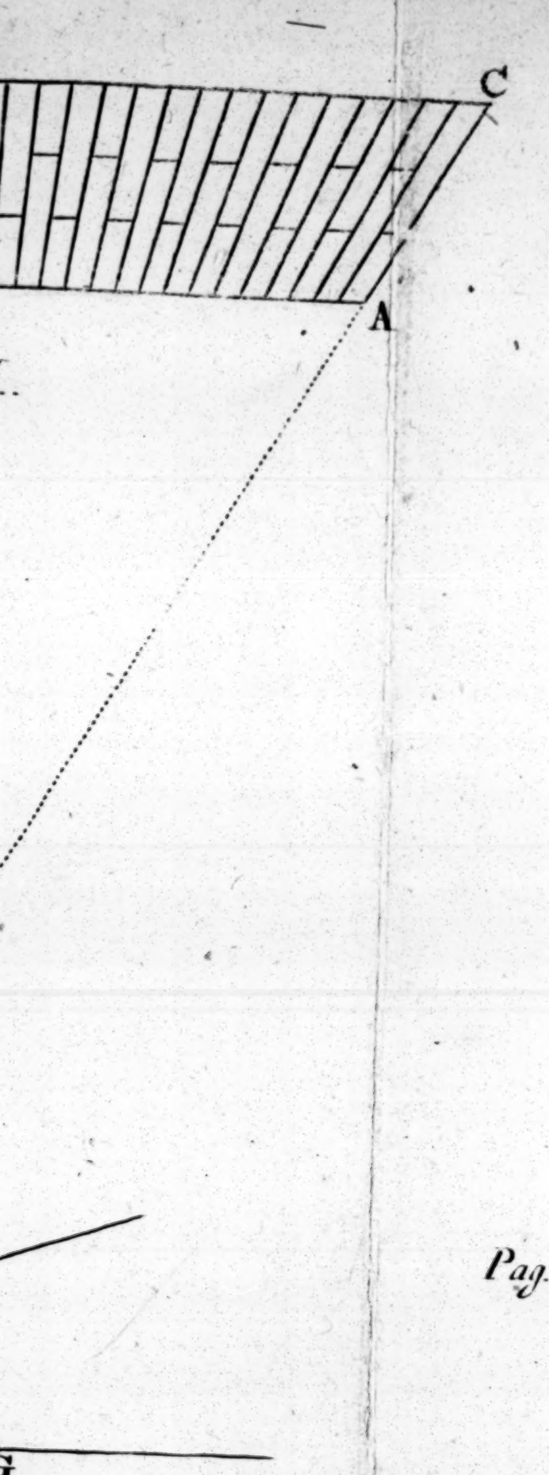


Fig XI



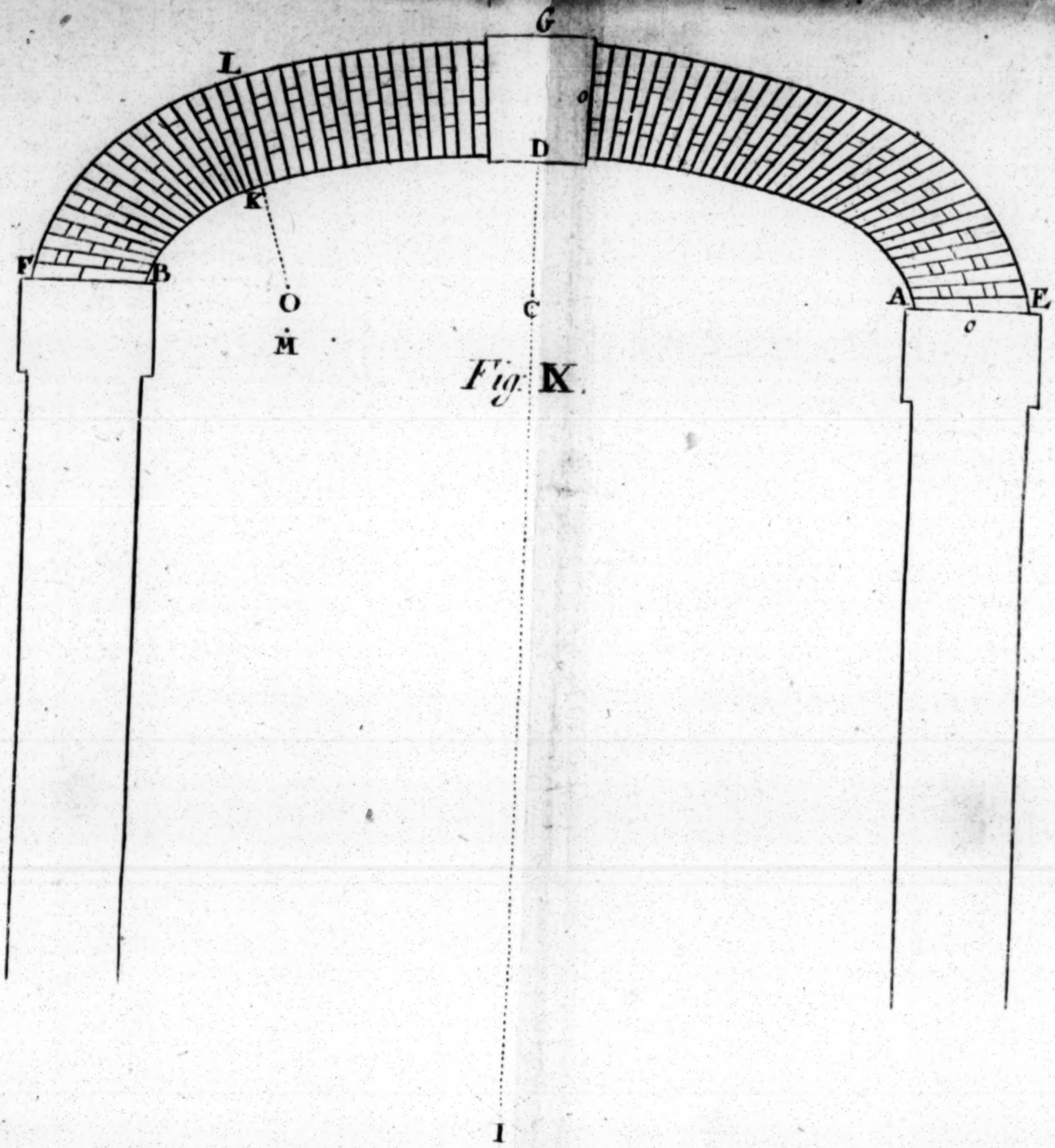


Fig. IX.

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Then take a Bevil, and laying the inner Edge of it ſtrait with the Line A B, and the Angle of the Bevil juſt over the Angle at A; take off the Angle that the Skew-back Line A C makes with the Line A B, and ſet it upon the ſmoothed Side of your Sommering Mould for the Bevil Stroke of your firſt Courſe; then drawing your Bevil towards E, ſtrait in the Line, until the Angle of the Bevil be juſt over the Angle that the ſecond Sommering Line makes with the Line A B. When it is ſo, draw the Tongue of the Bevil to lie even upon the ſecond Sommering Line, (in brief, cauſe the Bevil to lie exactly on the Line A B, and on the ſecond Sommering Line) then take up your Bevil, and lay it on the Mould, and ſtrike that Bevil Line on the Mould with the Point of the Compaſſes about half a quarter of an Inch diſtant from the firſt, and that is the Bevil of the Under-ſide of the ſecond Courſe. Proceed thus, until you come to the middle Line E F, but after you have ſet 3 Bevil Lines upon your Sommering Mould, leave about $\frac{1}{4}$ of an Inch between the 3d and 4th, and ſo likewiſe between the 6th and 7th, and the 9th and 10th; which will be a great Help to you in knowing the Number of each Line on the Mould.

The Moulds for the other half of the Arch, namely E B, are made after the ſame Manner; but the Arch is deſcribed from a Centre beneath P, as Q, which cauſeth a leſs Skew-back (*viz.* B D.)

The diminiſhing of the Sommering Mould to any Skew-back may be found by the Rule of Three, by dividing a Foot into 10 equal Parts, and each of thoſe into 10 Parts, ſo that the whole Foot may contain 100 Parts. Then proceed thus: The upper

Line CF, will be 309, that is 3 Feet and almoſt 1 Inch, and the lower Line A E will be 252, that is, 2 Feet and an Half and $\frac{2}{100}$; and the upper Part of the Sommering Mould will be 17 almoſt, that is, two Inches of ſuch whereof there are 12 in a Foot; having theſe three Numbers (*viz.* 309, 252, 17) work according to the Rule of Three, and you will find 13 and $\frac{6}{7}$ of 100 Parts, that is almoſt 14 (ſuch Parts whereof there are 100 in a Foot, Line-meaſure) for the Breadth of the lower Part of the Mould.

You may likewiſe find it Geometrically thus:

Having drawn the upper Line and under Line of the Arch, as C F and A E, and drawn any Skew-back, as ſuppoſe A C in [*Fig.* X.] make at Diſcretion the Angle G C H in [*Fig.* XI.] then take the upper Line C F, and ſet it from C to F; alſo take the lower Line A E, and ſet it from C to E, and draw the Line E F; then take the Thickneſs of your Brick, which ſuppoſe to be 2 Inches, and ſet it from F to G, and draw G H parallel to F E; I ſay, F G is the Breadth of the upper Part of the Sommering Mould, and E H the Breadth of the lower Part. Then make your Sommering Mould true to thoſe two Lines, and beginning in the middle Line A C, deſcribe the ſtrait Lines by the Mould from the Key F E, until you come to the Skew-back A C, and then take off the Bevil Lines, and ſet them on your Sommering Mould: With which I conclude this firſt Book of Geometry, being as it were, an Introduction to that which follows

A

GARDEN

OF

Geometrical Roſes:

OR, SOME

PROPOSITIONS

Being hitherto hid, are now made known.

The SECOND BOOK.

Written in *Latin*

By *THOMAS HOBBES*:

And done into *Engliſh*

By *VEN. MANDEY*.

LONDON:
Printed in the Year M.DCC.XXVII.

Geometrical ROSES.

PROP. I.

Of cutting a Right Line in extream and mean Proportion.

LET there be deſcribed a Square A B C D, and let each Side be divided in the Middle, in E, F, G, H; and F E, G H, being joined, they will cut each other in the Centre of the Square at I. Likewiſe from the Centre D, let there be deſcribed a Quadrant D A C, cutting F E and G H in K and X.

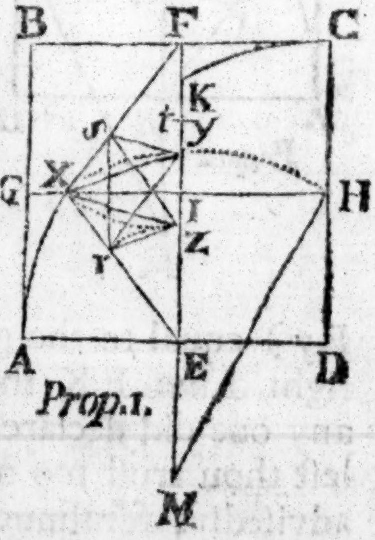
Laſtly, Let E X be drawn; I ſay E X is equal to the greater Segment of the right Line E F, or of the Side A B, being divided in extream and mean Proportion.

Let F X be drawn, and with that Semidiameter deſcribe an Arch of a Circle X z, cutting F E in z. Likewiſe with the Semidiameter E X, deſcribe an Arch of a Circle X y, cutting the ſame F E in y, and let the right Lines X z, X y be drawn.

Now the Angle E z X is equal to thoſe 2 Angles z F X, F X z, becauſe theſe two Angles are within, the other is without the Triangle zFX. Again, the ſame Angle EzX for the ſame Cauſe is equal to the 2 Angles Xyz, yXz. And Xzy and FXz are equal. Wherefore alſo the 2 Angles XFz, and zXy, are equal to one another.

Therefore the 2 Triangles FXz, yXz, have 2 Angles equal to 2 Angles of the Triangle zXy, and by conſequence 3 to 3: Therefore they are Equiangled. Wherefore, as XF or EX, to Xz; ſo is Xz or Xy, to zy. Therefore Xz (or Xy) is a mean Proportional between EX (or Ey) and zy.

In the right Lines EX, FX, let there be taken Er, Fs, and either of them equal to Ez, or Fy. Let rs, and sz be joined. Then rX, Xs will be equal. Alſo rs will be parallel to the right Line EF; and therefore the Altern Angles rsy, Fys, and likewiſe the Altern Angles sry, Ezr, are all equal to one another; and Fsyr, and Ezsr, will be Rhombus's both alike. Therefore the right Lines Fs, sz, ry, rE, Ez, Xy, are equal to one another.



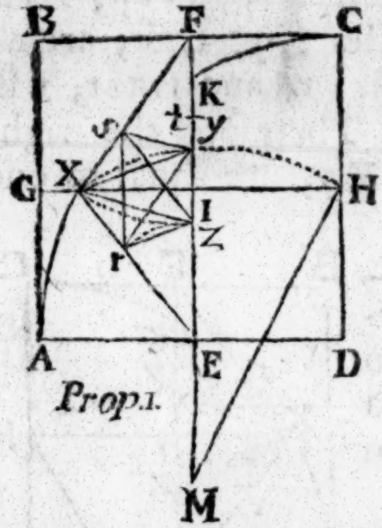
*Prop.*1.

And becauſe it is manifeſt, that Xy is a mean Proportional betwen Ey and zy; alſo Fy (equal to Xy) will be a mean Proportional between Ey and zy.

Therefore it will be, as Ey to Fy, ſo Fy to zy; and being compounded, as Ey more yF (that is EF) to Fy more yz (that is, Fz, or Ey) ſo likewiſe is Ey to Ez. Therefore EF is divided in z and y, ſo that the whole Line EF, a Part Ey, and the remaining Part Ez; likewiſe FE, a Part Fz, and the remaining Part Fy, are continual Proportionals. Therefore Ey, that is EX, is the greater Segment of the right Line EF, being divided in extream and mean Proportion, which was to be demonſtrated. Let him reprehend it that can.

ANIMADVERS.

To him that rightly conſiders, it is a Paradox: Neither *Euclid*, nor any other, hath taught this. Who ever thought that right Line which joins half one Side of the Square, with one third Part of the Quadrant inſcribed within the Square, to be the greater Segment of the Side cut in extream and mean Proportion? Thoſe Segments, of all hitherto, have been deſigned after this Manner. Produce the right Line IE in M, ſo that IE, EM, may be equal, and IM equal to the Side. Moreover from the Centre M, with the Radius MH or MG, deſcribe an Arch of a Circle, it will cut off a Part from EF (ſuppoſe Ey) equal to the greater Segment. But that the right Line EX ſhould be equal, they never, nor any one did declare. Therefore, Reader, beware, leſt thou truſt too much in an uncertain Thing unadviſedly, foraſmuch as we take Things near the Truth



Prop.1.

Truth for true Things. Examine the Demonſtration, and confer with true or ſurd Numbers, or conſult Algebraiſts.

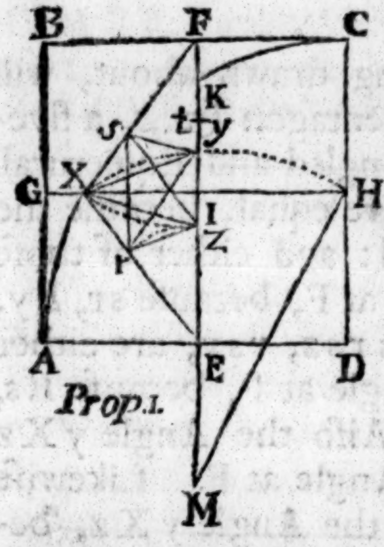
CONSECT. I.

THE Lines sy, rz, being drawn about, will make the Figure of a Pentagon (*viz.* a five-ſided Figure) Xsyzr, Equiangled and Equicrural. For the Angles Xsr, Xrs, are equal, becauſe the right Lines Xs, Xr are equal: and either of thoſe Angles are equal to the Angle at F, becauſe sr, Fy, are Parallels; and the Angles rsz, rsy, are either of them equal to the ſame Angle at F, becauſe Es, Fr, are Rhombus's alike. Alſo the Angle yXz appears to be equal to the Angle at F. Likewiſe the Angle Xzr, is equal to the Angle yXz, becauſe rz, Xy are Parallels. Laſtly, the Angles Xzs, Xyr, are equal to the ſame Angle yXz, becauſe the Baſes Xs, Xr, yz, are equal.

CONSECT. II.

THE Square of the greater Segment Ey, is equal to two Squares (to wit) to the Square of the right Sine of 30 Deg. and to half the Square of ſo many Deg. of the Line of Chords. For EI is equal to the right Sine of 30 Deg. And becauſe (the Chord XK being drawn) the Triangle XKI, is made Equicrural and right Angled, the Square of XI will be half the Square of the Chord XK; therefore Ey, that is, EX being ſquared, contains as much as the Square of EI and XI added together.

CONSECT. III.



Ky is the Half of FI: cut FI in the Middle at t; therefore becauſe Fy more yI, and It more tF are equal; by how much Fy is greater than Ft, by ſo much tI is greater than yI; but Fy is greater than Ft, by ſo much as ty is.

Wherefore alſo tI, that is Ft, is greater than yI, by the ſame Quantity ty. And Ft is divided in K, as tI in y. For becauſe Fy more yI, and IK more KF are equal; by how much Fy is greater than IK, ſo much is FK greater than yI.

Therefore when as Ft is made equal to ty more yI; Ft will be divided in K, as tI in y.

Wherefore tK is equal to yI; and Ky equal to FK more yI. Therefore Ky is equal to the Half of FI.

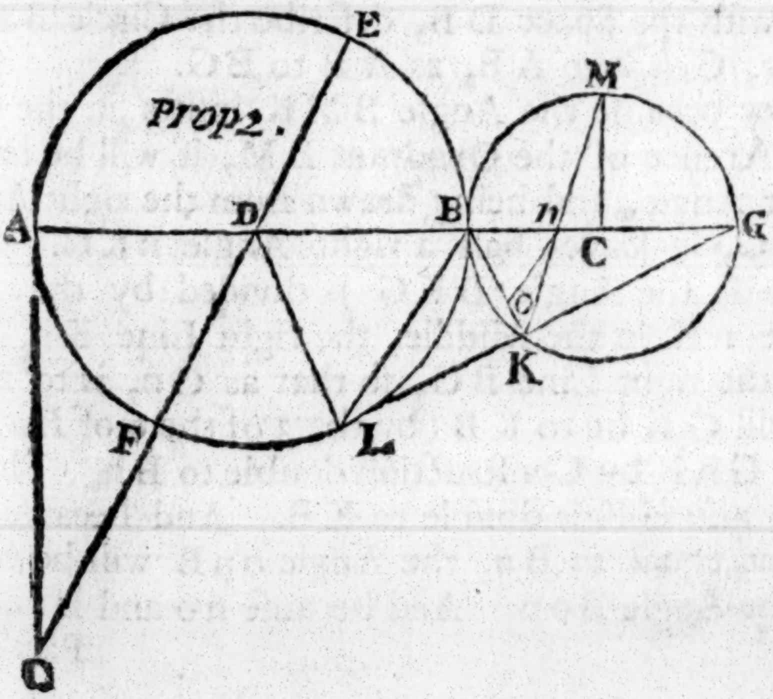
PROP.

PROP. II.

One Segment being given of a right Line divided in extream and mean Proportion, to find the other.

LET the right Line given be A B, being the greater Segment of any right Line. Cut A B in the Middle in D, and from the Centre D, with the Interval (or Semidiameter) A D, deſcribe the Circle A F B E. Then to the Point A, raiſe a Perpendicular A C, equal to the given Line A B; and through the Centre D, draw a Line to the Concave Periphery C E, to which let A G be made equal.

I ſay, the whole Line A G, hath the ſame Proportion to A B, that A B hath to B G. For, be-

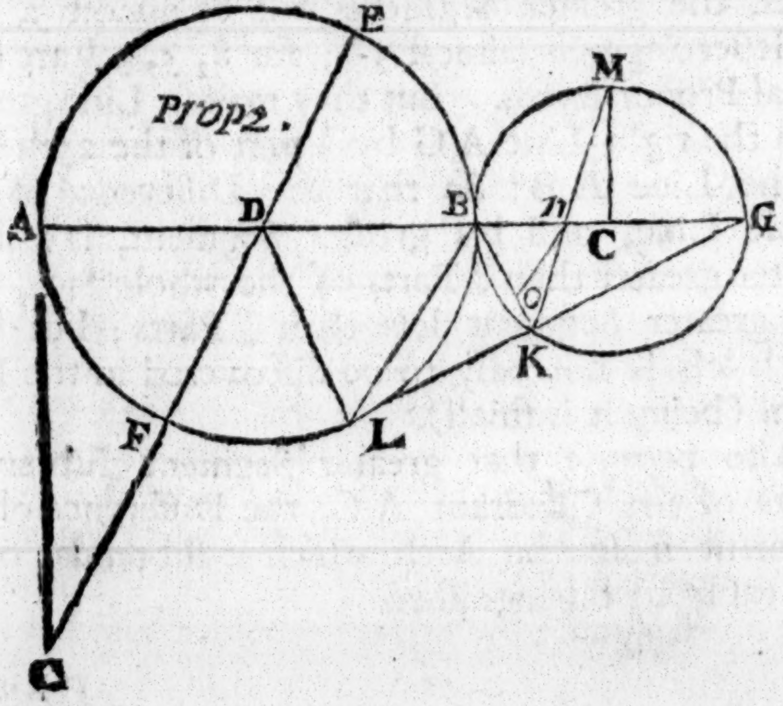


cauſe (by the 36 of the 3d Book of *Euclid*) as E C, is to E F; ſo is E F, to F C. Alſo as A G, together with his Equal E C, is to A B, and his Equal E F; ſo will A B, be to B G, and his Equal F C. Wherefore (by the Definition of extream and mean Proportion) the right Line A G is divided in extream and mean Proportion in the Point B.

Again, let B G be the leſſer Segment of any right Line. Cut B G the given Line in the Middle in C. And from the Centre C, with the Semidiameter C B, deſcribe a Circle B M G. In which Circumference, the Quadrant B M being taken, draw by the Point n, the right Line M K, ſo that B n be a third Part of the given Line B G. Then draw B K and G K, and in the Line B K, take a Part B o, which ſhall be equal to the right Line B n, draw the Line o, n, to which let there be a parallel Line drawn, B L, cutting G K being produced in L. Finally, raiſe a Perpendicular to G L in the Point L, which will cut G B, being produced in D. From the Centre D, with the Space D B, deſcribe the Circle B L A. I ſay, G A is to A B, as A B to B G.

For becauſe the Angle B M K inſiſts in the Circumference of the Quadrant B M, it will be half a right Angle, and being drawn from the right Angle B K G, it leaves half a right Angle n K G. And becauſe the Angle B K G is divided by the right Line n K in the Middle, the right Line K n, will cut the right Line B G, ſo that as G n, is to n B; ſo will G K be to K B (by the 3 of the 6 of *Euclid*). But G n is by Conſtruction double to B n Therefore alſo G K is double to K B. And becauſe B o is put equal to B n, the Angle o n B will be equal to the Angle B o n. And becauſe n o and B L, are

Parallels, the Angle L B D will be equal to the Angle o n B. Likewiſe becauſe L D and K B are Parallels and Perpendiculars, the Angles D L B, and B o n, will be equal. Likewiſe B o n, and B n o, are equal. Therefore D L B and B n o are equal. But B n o and D B L are equal, therefore D L B and D B L are equal. Therefore alſo the right Lines D L, D B, being Chord Lines, are equal. Likewiſe the Circle deſcribed with the Space (or Semidiameter) D B will paſs by the Point L. And becauſe D L is perpendicular to G L, G L toucheth the Circle B L A in L. Therefore it will be (by the 36 of the 3 of *Euclid*) as G A to G L, ſo G L to G B. And A B is equal to the ſaid G L. (For whereas the right Line G K is double to the right Line B K; ſo alſo the right Line G L, will be double to the right Line L D). Therefore in the



ſame Manner the right Line B A is double, and for that Cauſe equal to the right Line G L. Therefore alſo it will be as G A is to A B; ſo A B to B G, and by conſequence, we have added the other Segment of the right Line given, &c. which was to be done.

ANIMADVERS.

I ſee not wherefore he hath invented this, new Way of cutting proportional Lines, unleſs, perhaps, from this he thought to find the Greatneſs of the Circle, which he ſhould ſeek for, by comparing the Power of the greater Segment, with the Power of the Semidiameter, not before known.

The Quantities of theſe Segments, nor the Squares of them, cannot be expreſſed accurately by Numbers. But their Proportion comes very nigh to the Proportion of 5 to 3; for if the Side A B be 8 Parts, the greater Segment will be almoſt 5, and the leſſer Segment almoſt $3\frac{1}{8}$; for 8, 5, $3\frac{1}{8}$ are continual Proportionals. But they make a Line greater than the right Line A G by $\frac{1}{8}$ part of the 25th Part of the Line A B: So that the Difference of the whole Line, and his greater Segment, is a ſmall matter greater than $\frac{3}{8}$ Parts of the whole Side, and the greater Segment leſs than $\frac{5}{8}$ Parts; but how much leſs is not eaſy to be diſcovered in the Diagram (being it is ſmall).

Alſo becauſe that greater Segment ſubtends $\frac{2}{5}$ Parts of the Quadrant A C, the Difference of the Segment from the Arch which it ſubtends, could not eaſily be diſtinguiſhed.

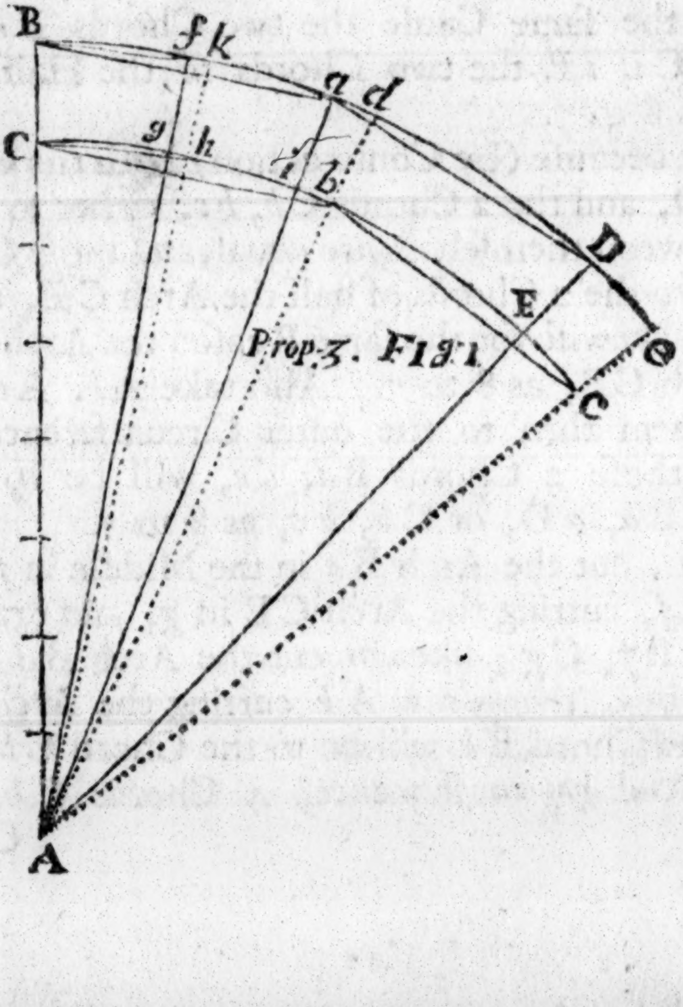
PROP.

PROP. III.

Of Regular POLYGONS.

In a Circle given to deſcribe a Regular Heptagon.

WHatſoever right Line, ſuppoſe A B, let it be cut in 8 equal Parts, whereof let A C be 7. Then from the Centre A, with the Semidiameters A B, A C, deſcribe 2 Circles. Moreover take one eight Part of the Perimeter of the outward Circle (to wit) B D (which is eaſily done) and draw the Line



Prop. 3. Fig. 1.

Line A D, cutting the inner Circle in E, which will cut off the eighth Part of it C E.

Divide the Arch B D in the Middle in *a*, and draw the Chord Lines B *a*, *a* D; likewiſe in the Arch C E apply the right Line C *b* equal to B *a*, and again *b c* equal to the ſame B *a* or *a* D; for they are equal.

I ſay, A right Line C *c* being drawn, is the Side of a Heptagon in the Circle C E. For becauſe A B is to A C, as 8 to 7; ſo alſo the Perimeter of the Circle B D to the Perimeter of the Circle C E, will be as 8 to 7.

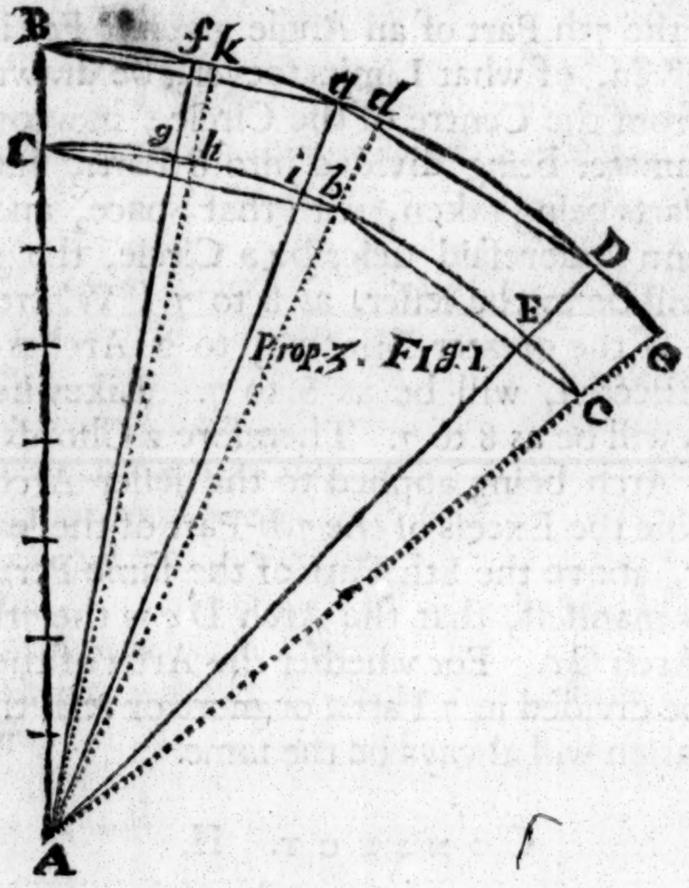
Alſo becauſe the Sectors A B D and A C E are alike, and the Triangles A B D, A C E are alike, both the Arch B D, to the Arch C E, and the Chord B D, to the Chord C E, will be as 8 to 7.

For the ſame Cauſe the two Chords B *a*, *a* D will to C *i*, *i* E the two Chords of the Half Arch C E, as 8 to 7.

Now becauſe (by Conſtruction) both the Chords B *a*, *a* D, and the 2 Chords C *b*, *b c*, either to other, and between themſelves, are equal, and thoſe Chords will be to the 2 Chords of half the Arch C E, as 8 to 7. And likewiſe for the ſame Reaſon the Arch C *c* to the Arch C E, as 8 to 7. Alſo take A *b*, A *c*, and draw them thro' to the outer Circumference in *d* and *e*, thoſe 2 Chords B *d*, *d e*, will be to the 2 Chords B *a*, *a* D, or C *b*, *b c*, as 8 to 7.

Again, cut the Arch B *a* in the Middle in *f*, and draw A *f*, cutting the Arch C E in *g*, and draw the Chords B *f*, C *g*; likewiſe cut the Arch B *d* in the Middle in *k*, then draw A *k* cutting the Arch C E in *h*, the Chord B *k* will be to the Chord C *h*, as 8 to 7. And by conſequence, 4 Chords C *h*, to 4

Chords C g, as 8 to 7. Likewiſe if 2 Arches C *b* and C g, be cut in the Middle, and their Biſegments cut *ad infinitum*, every Chord of thoſe, to evey Chord of theſe, will be in Proportion, as 8 to 7. But the Chords thus taken infinite in any Arch are (all together) the Chord of the ſame Arch. Wherefore the Arch C *c* is to the Arch C E, as 8 to 7. And the Arch C *c* is the 7th Part of the whole Perimeter by C. Therefore a right Line being drawn C *c*, is the



Prop.3. Fig.1.

Side of a Heptagon in the Circle by C. Likewiſe the right Line A *e* being drawn, will cut the 7th Part of the Perimeter by B.

For if every one of the Chords to every one of the Arches which they ſubtend, are not equal, thoſe ſeveral Arches, and their Biſegments, may be again biſected, which is contrary to the Suppoſition. Therefore the Side of a Heptagon is found, which was propoſed.

CONSECT. I.

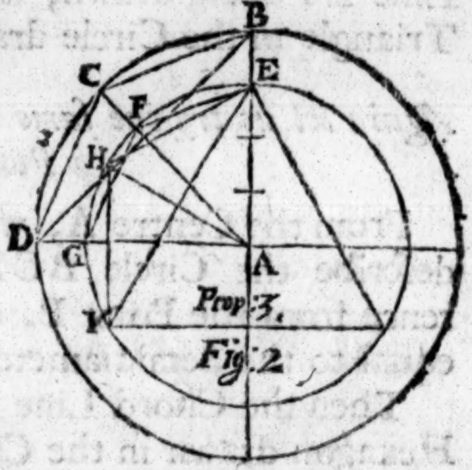
From this Demonſtration appears a Method of finding the 7th Part of an Angle given. For if to an Arch given, of what Limits ſoever, be drawn ſtrait Lines from the Centre of the Circle; moreover the Semidiameter being divided into 8 Parts, and 7 of thoſe Parts being taken, with that Space, and upon the Centre aforeſaid deſcribe a Circle, the greater Arch will be to the leſſer, as 8 to 7. Wherefore 2 Arches of the greater Biſected, to 2 Arches of the leſſer Biſected, will be as 8 to 7. Likewiſe their Chords will be as 8 to 7. Therefore 2 Chords of the greater Arch being applied to the leſſer Arch, will determine the Exceſs of the 7th Part of the leaſt Perimeter, above the 8th Part of the ſame Perimeter. For it is manifeſt, that the Arch D *c* is the 7th Part of the Arch C *c*. For whether the Arch of the Perimeter be divided in 7 Parts, or more or leſs, the Demonſtration will always be the ſame.

CONSECT. II.

2 An Arch being deſcribed with the Radius BC, is equal to the 8th Part of a Perimeter of a Circle whoſe Radius is AB: And to the 7th Part of the Perimeter of a Circle, whoſe Radius is AC: And to

a 6th Part of the Perimeter of a Circle, whoſe Radius is $\frac{6}{8}$ Parts of the right Line A B, &c.

Let us experience this our Method in known Polygons, and ſee whether or no by this, the Side of an Equilateral Triangle may be found from a Tetragon (*viz.* a Square).



Prop: 3.
Fig: 2

From the Centre A, deſcribe the Circle BCD, whoſe Quadrant is B D. Then draw the Chord Line B D, which is the Side of a Square inſcribed within that Circle.

From the ſame Centre A, with a Semidiameter A E (which let be to A B, as 3 to 4) deſcribe the Circle E F G; then the Circle by B, will be to the Circle by E, as 4 to 3.

Let the Quadrant B D be cut in two in the Middle in C, and let the equal Chord Lines B C, C D, be drawn. Then draw A C, cutting E G in F, the Arch E F will be the 8th Part of the Perimeter by E, and equal to the Arch F G.

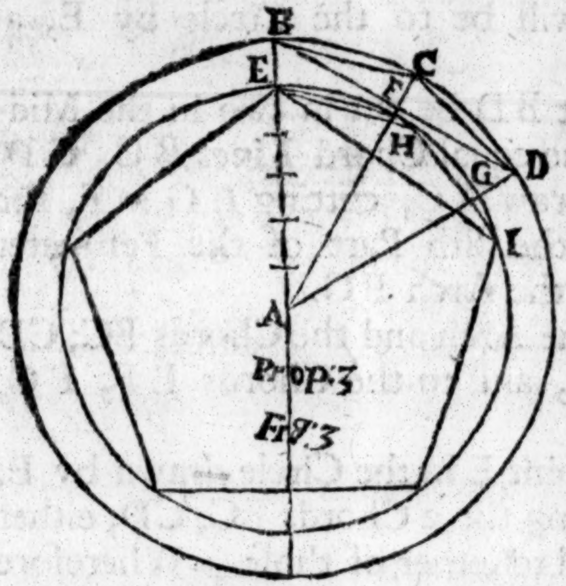
Therefore both the Arch and the Chords BC, CD will be to the Arch, and to the Chords E F, F G, as 4 to 3.

Then from the Point E in the Circle drawn by E, apply EH, HI, being the 2 Chords BC, CD, either of theſe being equal to either of thoſe. Wherefore the 2 Chords E H, H I, are to the 2 Chords E F, F G, as 4 to 3; that is, as the Arch BD to the Arch E G. Wherefore as the Arch E I to the Arch E G, ſo is 4 to 3. Therefore the Arch G I is the 3d Part of the Arch E G, that is, a 12th Part of the whole Perimeter drawn by E; and the Arch E I, a 3d Part of the ſame Perimeter. The Demonſtration is the ſame which concerns the Heptagon, which you have before in *Pag.* 50. Therefore the Chord Line E I being drawn, is the Side of an Equilateral Triangle in the Circle drawn by E.

Again let us try the ſame Method from a Hexagon to a Pentagon.

From the Centre A, with the Semidiameter A B, deſcribe the Circle B C D; and in that Circumference from the Point B, apply the Chord Line B D equal to the Semidiameter A B.

Then the Chord Line B D, will be the Side of a Hexagon drawn in the Circle by B.



In the Semidiameter A B, take A E, which muſt be to A B, as 5 to 6. And from the ſame Centre A, with the Radius A E deſcribe the Circle E F G. Then the Perimeter by B to the Perimeter by E, will be as 6 to 5. Let B D be cut by a right Line A C (cutting the Circle drawn by E, in F) in the Middle in C; and let the Chords B C, C D be drawn, equal to which apply the

Chords E H, H I, in the Circle drawn by E. Then are the two Chords E H, H I, to the two Chords E F, F G, as the Arch B D to the Arch E G, that is as 6 to 5; and the Arch E I, to the Arch E G, as 6 to 5; and the Arch G I is a 5th Part of the Arch E G, that is, the 30th Part of the Perimeter; and the whole Arch E I, 6 of thoſe 30 Parts (that is, a 5th Part) of the whole Perimeter drawn by E. The Demonſtration is the ſame as in the Heptagon. Therefore the Chord Line E I, is the Side of a Pentagon drawn in the Circle by E.

Cor. 1. Therefore the Side of a Pentagon may be found, without the Work of cutting the Semidiameter in extream and mean Proportion.

Cor. 2. As from the outward Circle to the inward, the Demonſtration hath proceeded hitherto; ſo likewiſe it may proceed from the inward Circle to the outward. As from a Triangle given, may be found a Quadrat (or Square) and a Pentagon from a Square, and a Hexagon from a Pentagon, and ſo of the reſt.

For to the Side of a Pentagon given E I, the 2 Chords E H, H I are given. Wherefore if to the Semidiameter A E, be added a 5th Part of the ſame Semidiameter (to wit) E B, and the Arch B D be deſcribed; the 2 Chords E H, H I, will be equal to thoſe 2 Chords B C, C D, and equal each to other, and the Chord B D, the Side of a Hexagon inſcribed in the Circle by B.

PROP.

PROP. IV.

Of the Proportion of crooked Lines, to crooked Lines in the Circumferences of Circles.

1. AS a Circle is deſcribed from a Semidiameter with one Foot of a Pair of Compaſſes being fixed, and the other Foot carried about; ſo alſo it may be underſtood to be deſcribed from a right Line given, being bent uniformly, that is, ſo as the Angles be always equal; from which certain Flexion or Bending, if the Angles be conceived to be infinite in Number, is deſcribed a Circle. For a Circle in its Nature, differs nothing from a Polygon of an infinite Number of Sides.

And bending is a departing from Straitneſs according to ſome Angle, which is Crookedneſs.

2. And the Crookedneſs of ſome to other ſome, is greater or leſſer; and therefore Crookedneſs is Quantity, and belongs to the Subject of Geometricians, and chiefly to thoſe who write concerning the Magnitude of a Circle, and the Inſcription of Polygons in a Circle, altho' concerning this Thing, nothing hath been delivered to us from the Antients.

3. What Proportion an Angle in a Circumference, to an Angle, in a Circumference in the ſame Circle hath, the ſame Proportion hath the Crookedneſs of the greater Arch, to the Crookedneſs of the leſſer Arch. For ſince Crookedneſs is no other Thing, than a Bowing (by an Angle in a Circumference) from Straitneſs; it muſt be that the greater Angle makes the greater Crookedneſs. From whence it follows, that the Crookedneſs of Arches in the ſame Circle, be to each other like their Angles.

4. In divers Circles the Crookedneſs of the greater Perimeter, is leſs than the Crookedneſs of the leſſer Perimeter, in the Proportion of the Radius to the Radius, or of the Diameter to the Diameter reciprocal. For in great Circles, as in the great Circle of the Earth, no Crookedneſs can be diſcerned in a long Space, but in a Ring it is every where diſcern'd; therefore in indifferent Circles, the leſſer Circles have the greater Crookedneſs, for this very Cauſe, becauſe the Diameter is leſſer; which is manifeſt by the Light of Nature.

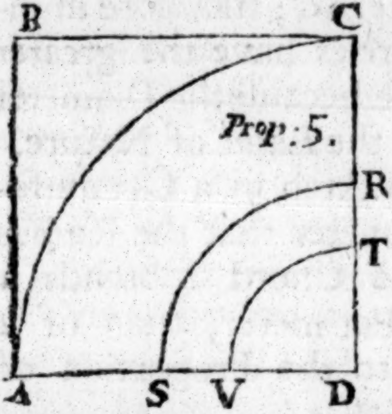
5. (If the Proportion of an Arch in a Circumference be the ſame to the Perimeter that the Radius is to the Radius) Any ſame Chord ſubtends a greater Portion of his own Perimeter, than of a greater Perimeter, according to the Proportion of the greater Perimeter to the leſſer.

For the Cauſe wherefore the ſame right Line, ſubtends a greater Part of the leſs Perimeter than of the greater, is the only and eſſential greater Crookedneſs of the ſame Line, being bowed in the leſſer Perimeter, than when it is bowed in a greater Perimeter: Therefore if the Crookedneſs of a leſſer Arch to the Crookedneſs of a greater Arch in a Semicircle be as 8 to 7, the Chord, which ſubtends the eighth Part of the greater Arch, will ſubtend the ſeventh Part of the Semiperimeter of the leſſer: I ſay in a Semicircle, becauſe the Crookedneſs of a Perimeter beyond a Semicircle proceeds a contrary Way, and becauſe the Subtenſes of the Arches, which together with themſelves in a Semicircle increaſe; but beyond a Semicircle they all decreaſe.

6. Alſo therefore an Angle in the Circumference of a leſſer Perimeter (becauſe the Subtenſes are equal) will be to an Angle in a greater Perimeter, as the greater Perimeter to the leſſer.

PROP. V.

A mean Proportional between the Semidiameter of a Circle, and $\frac{2}{5}$ Parts of the ſame is equal to $\frac{2}{5}$ Parts of the Fourth Part of the Circle.



DEſcribe a Quadrant of a Circle D A C, and compleat the Quadrat A B C D; in the Side D C take D T, being two fifth Parts of the Side D C, and between D C and D T, let there be taken a mean Proportional DR, ſo that DC, DR, DT, may be continual Proportionals; alſo deſcribe the Quadrantal Arches R S, T V; then the Arch T V is two fifth Parts of the Arch C A.

I ſay the Arch T V, and the right Line D R are equal.

Let it be ſuppoſed that there is a right Line given equal to the Arch A C, and from it deſcribe a Quadrantal Arch (*viz.* with the right Line given being Radius, deſcribe the fourth Part of a Periphery) the Line D C, C A, and the Quadrantal Arch above C A are continual Proportionals.

Let there be writ apart	D C, C A, the Arch above C A ∺
And under theſe	D R, R S, the Arch above R S ∺
Again under theſe	D T, T V, the Arch above T V ∺

Then the Antecedents to the Conſequents will be thro' all the Orders or Ranks forward, as the Semidiameter
]

midiamerer to the Arch of a Quadrant, and backward, as an Arch of a Quadrant to the Semidiameter.

Therefore as D C to R S, ſo is R S to the Arch above T V; therefore D C, R S, and the Arch above T V, will be continual Proportionals. And becauſe D C, C A, and the Arch above C A are likewiſe continual Proportionals, and have the firſt Antecedent D C common; the Proportion of the Arch above C A to the Arch above T V, will be (by the 28 of the 14 of *Euclid*) in Duplicate Proportion of C A to R S; and for the ſame Cauſe the Arch above R S, is a mean Proportional between the Arch above C A, and the Arch above T V. Now if D C be greater than R S, likewiſe R S will be greater than the Arch above T V; and the Arch C A greater than the Arch above R S. Wherefore, when as D C, C A, and the Arch above C A are continual Proportionals, the Arch above T V, and the Arch above R S, and the Arch above C A cannot be continual Proportionals, becauſe it is demonſtrated to the contrary. Therefore D C is not greater than R S.

Again let R S be ſuppoſed to be greater than D C; then the Arch above R S will be a mean Proportional between the Arch above T V, and the greater Arch above C A. Therefore the Inconvenience will return.

Wherefore the Semidiameter D C is equal to the Arch R S; and by Conſequence the Arch F V, that is two Fifths of the Arch C A, and the right Line D R are equal, that is, a mean Proportional between the Semidiameter and two fifth Parts of the ſame, is equal to two fifth Parts of the fourth Part of the Circumference of the Circle: Which was to be demonſtrated.

PROP. VI.

LET there be deſcribed a Quadrat A B C D, and let it be cut in the Middle both Ways by E F, G H; alſo let it be cut by the Diagonals A C, B D (concurring in the Centre I) four Ways.

Moreover, between D C and two fifth Parts thereof, let there be taken a mean Proportional D R, and join A R, cutting E F, in *a*, let it be produced until it meet with the Side B C being produced in *b*.

I ſay that the right Line B *b* is Quintuple to (*viz.* five times as long as) the right Line E *a*, or the fifth Part of the Arch A C (or it may be thus render'd, the right Line B *b* is equal to the Quadrantal Arch A C.)

Becauſe the Triangles A D R, A B *b* are alike, and the Arch of a Quadrant deſcribed from D R is equal to the Side D C, or A B; likewiſe the Arch deſcribed from A B will be equal to the right Line B *b*. Therefore whereas D R is two fifth Parts of the Arch A C, and by conſequence B *b* five of thoſe Fifths; B *b* will be five times as long as the right Line E *a*, which is the Half of the right Line D R, and the fifth Part of the Arch A C, or the right Line B *b*. Which was to be demonſtrated.

Therefore if in the Side B C, be taken a part *B i*, equal to E *a*, that part B *i* being five times repeated, will end in *b*.

A compendious Expoſition.

If 2 right Lines whatſoever have a mean Proportional between them, which is equal to the Side A B, it will be as the Arch A C to one of them; ſo reciprocally the other of them will be to two fifth Parts of the ſame Arch A C; as in Example, becauſe the Side A B is a mean Proportional between the Arch A C, and two Fifths of the ſame; and the ſame mean Proportional between the Diagonal Line A C, and the Half of it D I. The right Angle under the Arch A C, and two fifth Parts of the ſame, will be equal to the right Angle under B D, and the Half of it D I. Therefore as the Arch A C is to his Diagonal A C, or B D, ſo will the Half Diagonal D I be to D R. And as the Half of the Arch A C, to the Half of the Diagonal, ſo is the Half Diagonal to D R.

CONSECT.

From hence appears the Magnitude of the eighth Part of the whole Perimeter. For if to the right Line D R, be added R *z*, being a fourth Part of D R, the whole Line D *z*, will be five of thoſe ten Parts, that is half the Arch A C. For when as D R is two Fifths, that is, four Tenths, D *z* will be five Tenths.

PROP. VII.

THE ſame Lines continuing, draw the right Line D F. I ſay D F is a mean Proportional between the whole Arch A C, and his Half.

Draw the right Line R *r* parallel to the Side B C, cutting the Diagonal D B in r, and D F in *d*. Then becauſe D F cuts the Side B C in the Middle in F, the right Line D *d* will cut the right Line R *r* in the middle in *d*. Let L N be drawn parallel to the ſame Side B C, cutting D F in *e*, and the Side D C in N. Then D N and N L will be equal, and either of

them

them equal to the Semidiagonal D I, and N L, will be divided in the Middle in *e*.

Therefore becauſe by the compendious Expoſition of the precedent Prop. D *z*, D N, D R, are continual Proportionals: If with the Radius D *z* be deſcribed an Arch *z f*, it will cut the right Line N L in *e*; likewiſe if with the Radius D N be deſcribed an Arch of a Circle N I, it will cut the right Line R *r* in *d*.

Then deſcribe the Quadrantal Arch N I O, which (as it is ſhewn) will paſs through *d*. Then becauſe D R is the Radius of a Circle, whoſe fourth Part of the Perimeter is equal to the Side D C; the right Line D *d* or D N will be the Radius of a Circle, whoſe fourth Part of the Perimeter is equal to the right Line D F. But the right Line D N is the Radius of a Circle, whoſe fourth Part of the Perimeter is the Arch it ſelf NIO.

Therefore the right Line DF, and the Arch NIO are equal: And the Arch NIO is a mean Proportional between the Arch A C and his Half, therefore alſo the right Line D F is a mean Proportional, &c. which was to be demonſtrated.

Cor. If from a Point *z* be drawn a right Line *z c* parallel to the Side B C, cutting the Diagonal D B in *c*; D *c* will be equal to D F. For D C will be a mean Proportional between D *z* and the double of it. Likewiſe it is made appear, that D *z* is equal to half the Arch A C.

CONSECT. I.

The Arch A C, that is the right Line B *b*, is a mean Proportional between the Side D C, and the Quintuple of (*viz.* a Line five times as long as) the

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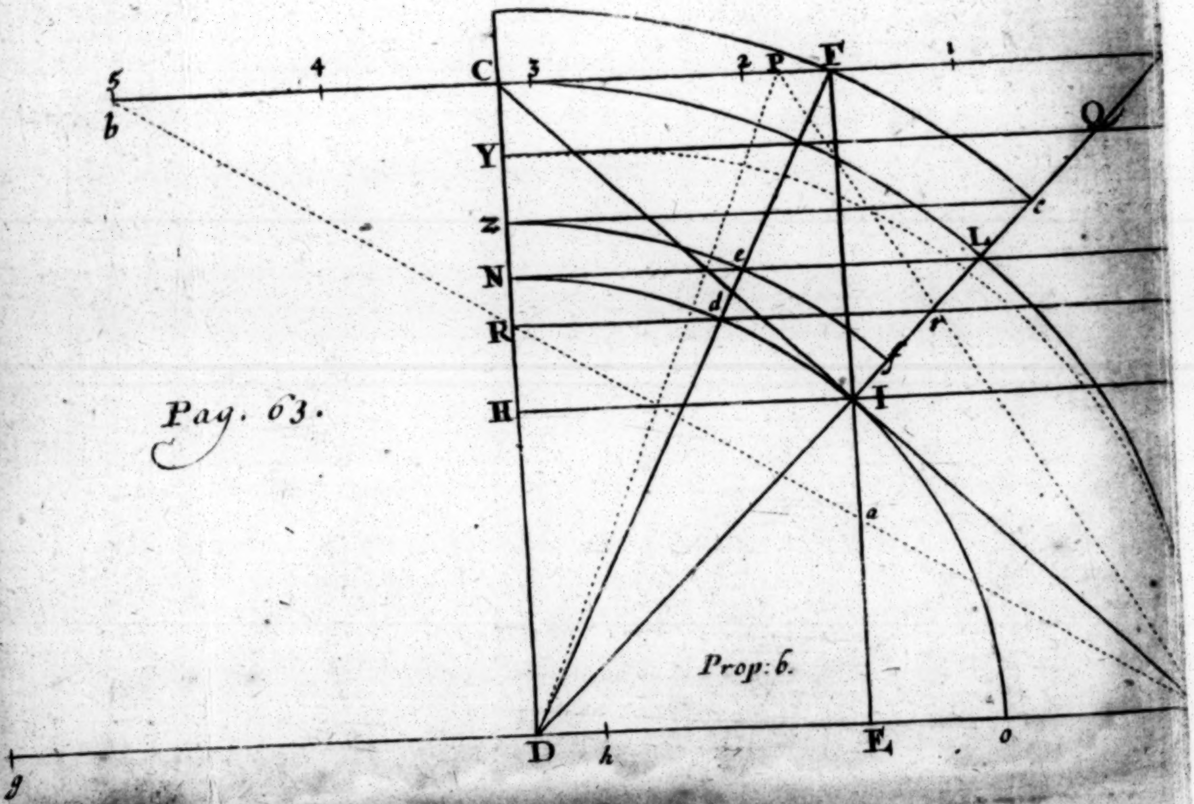


Prop: 6.

Half Side B F. For ſeeing that two Fifths of the Side D C, the right Line D R, and five of thoſe Fifths of the Side D C are continual Proportionals. If the Proportion be continued, D R, D C, and the mean Proportional between D C, and the Quintuple of the Half Side B F, will be continual Proportionals: Therefore a Quadrat being made or drawn from the right Line B *b*, is equal to ten Quadrates drawn from the Half Side B F. From whence alſo it follows, that a Quadrantal Arch being deſcribed from the Arch A C, being extended in a right Line, is Quintuple to the Half Side B F.

CONSECT. II.

The ſame Lines remaining, to the Side A D let there be added in a direct Line D *g*, equal to the right Line D *z*, that is equal to the Arch C L; moreover cut the whole Line A *g* in the Middle in *h*; then from the Centre *h*, with the Radius *h A*, or *h g*, deſcribe the Arch of a Circle A Y, cutting the Side D C in Y: Then D Y will be a mean Proportional between the Side D C and D *z*; and a right Line Y Q being drawn parallel to the Side B C, cutting the Diagonal D B in Q, Y Q will be the Side of a Quadrant (from *Archimedes*'s Demonſtration) equal to the Sector D C L, or an eighth Part of the whole Circle deſcribed from the Semidiameter D A.

CONSECT. III.

From hence it follows, that the Arch A C *is equal to the Compound of the Side* B C *and a Tangent of* 30 *Deg.*

For becauſe D C is a mean Proportional between the Arch A C and D R; if to D R and D C there be taken a third Porportional, it will be equal to the Side B C, together with the Tangent of 30 Deg.

But the Diagram muſt be renewed (or drawn anew.) Therefore let A B C D be a Quadrate, and let it be divided into four equal Parts by the right Lines E F, G H ; alſo let it be divided into four equal Parts by the Diagonal Lines A C, B D, concurring in the Centre I. Alſo draw the Quadrantal Arches A C, B D cutting E F and G H, in K and X.

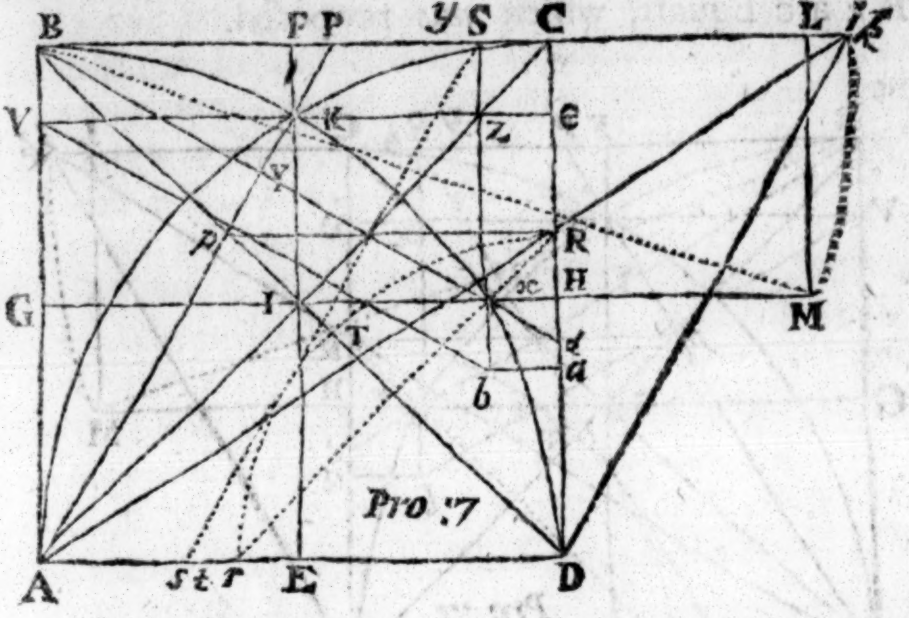
Produce the Lines B C, G H, and let there be taken the right Lines C L, H M, either of them being equal to half the Side, and let B M be joined. Then B M being taken as a Semidiameter, with it, from the Centre B, deſcribe an Arch of a Circle, cutting B C, being produced in *i*. Then becauſe B L being ſquared, contains as much as nine Quadrates (or Squares) made from the Half Side, and L M H C one of them Quadrates: Alſo the right Line B M, that is, B *i* being ſquared (to wit, a Quadrate being drawn with four Lines, each of them being the Length of B *i*) that Quadrate contains as much as ten Quadrates made from the Half Side C H ; and therefore B *i* is equal to the Arch A C, or B D.

Draw A K, and produce it to the Side B C in P ; A P will be a Secant of 30 Deg. and B P a Tangent of 30 Deg. and A P the double of B P. To the Side B C add C *k* equal to B P, and the whole Line B K will be compoſed of the Side B C, and a Tangent of 30 Deg. B P or C *i*.

It remains therefore to be demonſtrated, that *i* and *k* meet in the ſame Point.

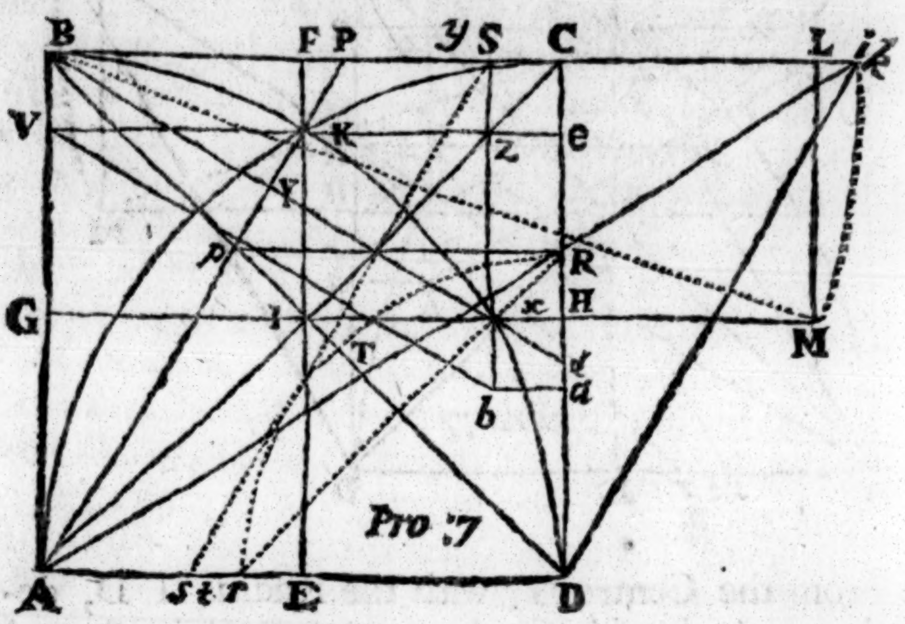
Let A *i* be joined, cutting D C in R ; and with the Radius D R, deſcribe a Quadrantal Arch R *r*, cutting the Diagonal Line B D in T.

From



From the Centre T with the Radius T D, deſcribe an Arch of a Circle cutting B C in S, and produce S T to the Side A D in S; then are D R, D T, T S Equals.

Then becauſe A B is a mean Proportional both between A P and E K, and between B *i* and D R, likewiſe between the Diagonal B D and its Half C I; it will be as B *i* or the Arch A C) to B D, ſo C I to D R or S T. And likewiſe as B D to A P, ſo reciprocally E K to D R or S. T. Wherefore if there be taken in the Side BC, a certain right Line equal to E K, ſuppoſe the right Line B *y*, and from thence to the Side A D be drawn *y t* parallel to A P, it will be as B *i* to *y t*, ſo B *y* to S T. Wherefore the right Line *y t* will paſs by T, and *y* T will be equal to S T, which is abſurd. Then B S is equal to E K, and S *s* equal to A P. Therefore becauſe A P is equal to D K, D K will be parallel to S *s*. Then as B *k* to D *k*, ſo is B S to S T: Therefore B *i* and B *k* are Equals, which was propoſed.



CONSECT IV.

From hence it follows, that the right Line H R (which is the Difference whereby two Fifths of the Arch A C exceeds the Half Side D H) is equal to F K the verſed Line of 30 Degrees.

For if in the Side A B, there be taken A V equal to E K, and with the Radius A V be deſcribed an Arch of a Circle, cutting A P in Y, the Arch V Y will be equal to the Half Side B F or B Y, and A Y equal to A V; and therefore B Y will paſs to *x*, and the ſame B Y will be a Tangent of 30 Degrees in the Circle, whoſe Semidiameter is A V or E K. Then produce V K to the Diagonal A C in *z*, V *z* will be equal to E K, wherefore a Perpendicular being dropt down from the Point S, it will paſs through *z* and *x*; and B *x* being produced to the Side D C in *d*, C *d* will be equal to C *i*: But S *x* is equal to half the Side, and therefore *z x* equal to K *z*.

In the Side C D, let there be taken the right Line C *a* equal to D R, and let *a b* be drawn parallel to the Side B C, cutting *z x* produced in *b*, and let V *b* be joined. Then becauſe as B *i* is (that is B C more the Tangent C *i*) to the Secant B *d*, ſo is B S to D R, that is to S *b*; and as B C to the Tangent C *d*, ſo V *z* to S *x*; if in S *x* produced, be taken *z b* equal to S *x*, *z b* will be a Tangent of 30 Degrees in the Arch deſcribed from V *z*.

Then join V *b*, it will be equal to the Side B C, and becauſe *z b* is a Tangent of 30 Degrees in the Circle, whoſe Radius is V *z*, and C *d* a Tangent of 30 Degrees in the Circle, whoſe Radius is A B; B *d* and V *b* will be Parallels; and join V *b* equal to the Side B C, and B C more C *d* will be equal to the right Line B *i*.

Then becauſe it is as B C more C *d* (that is B *i*) to B *d*, ſo V *z* to D R; and as the ſame B *i* to V *b*, ſo V *b* to D R, and S *b* will be equal to D R: For theſe two Analogies can no way be conſtituted in any other Point of the right Line S *x*.

Then the Equals D H being taken from D R, and *z b* from S *b*, there remains H R, S *z*, both Equals; but S *z* is equal to F K: Wherefore H R, F K are Equals.

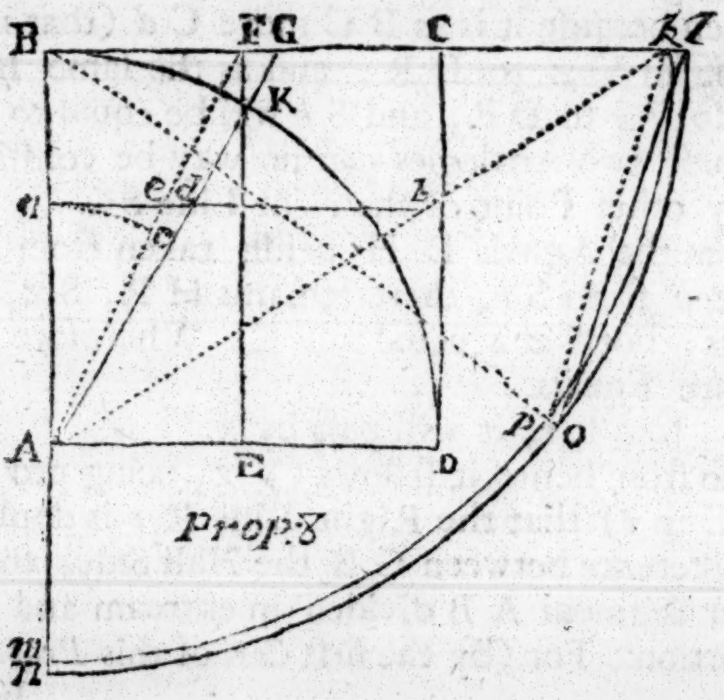
Cor. Join R *r*, it will paſs by *x*.

Alſo from hence it follows (V *z*, being produced to D C in *e*) that the Right Line R *e* is double to the Difference between G B the Half Side, and the greater Segment A B divided in extream and mean Proportion: For (by the firſt *Cor.* of this *Prop.*) the

Fourth Part of the Side A B, or D C, is equal to the right Line F K, together with the Difference between the Half Side, and the greater Segment of the Side: Wherefore half the Side, that is, *a e*, is equal to the double of F K, and to the double of the Difference between the greater Segment of the Side and the Half Side: But *a* R is double to F K; wherefore R *e* is double to the Difference between the greater Segment of the Side and the Half Side.

PROP. VIII.

AGain deſcribe a Quadrate A B C D, alſo deſcribe a Quadrantal Arch B D; and to the Side B C, let there be added C *k* equal to a Tangent of 30 Degrees, and let the right Line A *k* be drawn; moreover in the Side A B, let there be taken A *a* equal to the



the right Line A *c* being made equal to the greater Segment of the Side A B; (divided in extream and mean Proportion) then draw *a b* parallel to the Side B C, cutting A *k* in *b*.

I ſay, the right Line *a b* is the Subtenſe of two fifth Parts of the Quadrantal Arch *k m*, deſcribed with the Semidiameter B *k*, and thoſe two Fifths equal to the Side A B.

For becauſe A *a* is the greater Segment of the Side B C, being divided in extream and mean Proportion, it is alſo the Side of a Decagon in a Circle, whoſe Semidiameter is A B (by the 4 *Prop.* of the 14 *Elem.* of *Euclid*) and ſubtends the tenth Part of the whole Perimeter, that is, a fifth Part of the Half Perimeter, that is, two fifth Parts of the Arch B D.

Therefore ſince it hath been ſhewn, that the right Line B *k* is equal to the Arch B D, it will be, as A B to A *a*, ſo B *k*, (that is, the Arch B D) to *a b*.

Wherefore *a b* is the greater Segment of the right Line B *k* divided in extream and mean Proportion.

Apply to the Arch *k m* the right Line *k p* equal to *a b*; then the Arch *k p* will be two Fifths of the Quadrantal Arch *k m*, which is the firſt. But the Arch *k m* (by the 4 *Corol.* of *Prop.* 7.) is equal to five Half Sides of a Quadrate from A B; wherefore the Arch *k p* is equal to the Side A B, which remained to be demonſtrated.

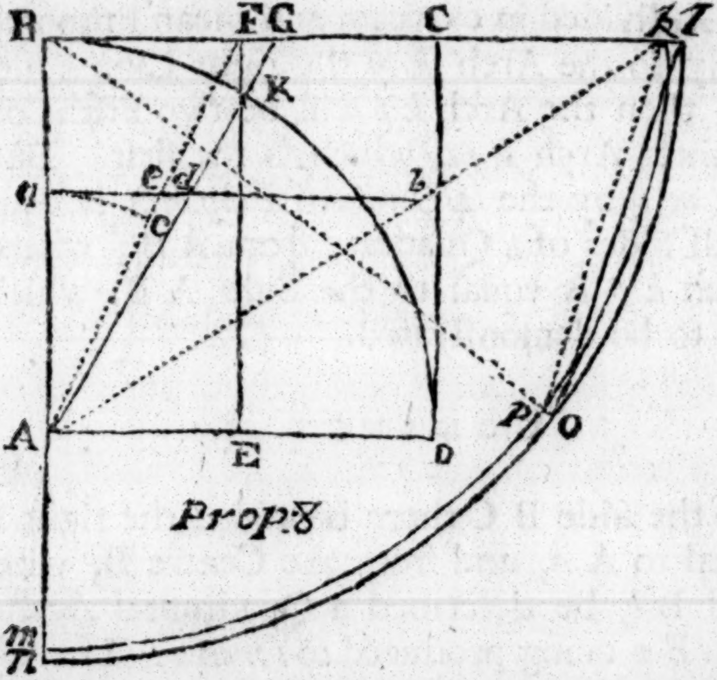
CONSECT. I.

If to the Side B C there be added the right Line C *l* equal to A *a*, and from the Centre B, with the Interval B *l*, be deſcribed a Quadrantal Arch *l n*, alſo join B *p* being produced to *l n* in *o*: The Chord

Line *l o* being drawn, it will be equal to the Arch *k p*, or the Side A B. For (by the 5 *Prop.* of the 13 *Elem.*) the Side B C will be the greater Segment of the whole B *l*: Wherefore the Side B C ſubtends two Fifths of the whole Arch *l n*. Therefore ſince the Angle *k* B *p* is two Fifths of the Angle *k* B *m* likewiſe *l o* will be two Fifths of the Arch *l n*: Wherefore the right Line *l o*, which ſubtends the Arch *l o*, is equal to the Side BC, and the ſame equal to the Arch *k p*.

CONSECT. II.

From hence it is manifeſt, that both the Quadrate it ſelf ABCD, and all the right Lines from the Centre A to the Side B C (if the Work be produced) are cut from *a b* (if the Work be produced) in



in extream and mean Proportion: For the Quadrate ABCD is divided in the ſame Proportion whereby the Side A B is in *a*; likewiſe from the ſame *a b* are divided all the right Lines drawn from A to B C (when the Work is produced) in the ſame Proportion whereby A B was divided in *a*: Wherefore (by the 2 Prop. of the 14 Elem.) they are divided in extream and mean Proportion.

Cor. From hence a manifeſt and brief Method appears of finding proportional Segments of right Lines of what Kind ſoever. As for Example; If the greater Segment of a Secant of 30 Degrees be ſought, draw the Line A K, and let it be produced to the Side B C in G, which is the Secant of 30 Degrees; it will cut the right Line *a b* in *d*, and A *d* is the greater Segment, and the Reſidue is the leſſer Segment: Or elſe the Section of a Tangent of 30 Degrees being ſought, which as aforeſaid is half the Secant, and the Side of a Cube inſcrib'd in a Circle, whoſe Diameter is A B.

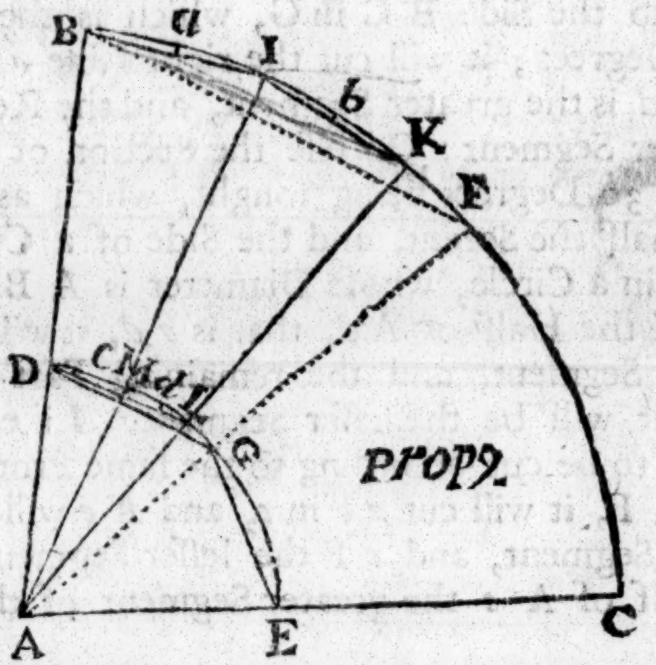
Take the Half of A *d*, that is *a d*, it will be the greater Segment, and the remaining Part of the Tangent will be the leſſer Segment. Likewiſe, if A F be to be cut according to the ſame Proportion, draw A F, it will cut *a b* in *e*, and A *c* will be the greater Segment, and *e* F the leſſer Segment; and the Half of A *e* the greater Segment of the Half of A F.

PROP.

PROP. IX.

To cut an Angle given, into any Proportion given.

LET the Angle given be B A C, and let the Proportion given be as A B to A D. Let the Arch A D E be deſcribed, and cut the Arch B C in the Middle in F. Wherefore A F being drawn, it will alſo cut D E in the Middle (ſuppoſe) in G. Then the Chords B F, D G being drawn, they will be between themſelves as the right Lines A B, A D.



Appſy the two Chords D G, G E, to the Arch B F in I and K, ſo that the Chords B I, I K may be Equals to the Chords D G, G E, and either to other; alſo draw the Lines A I, A K, of which A K cuts the Arch DE in L, and AI cuts the ſame Arch in M.

I ſay the whole Arch BC being thus divided in K, that the Arch BC (or the Angle given B A C) is to the Arch BK (or to the Angle BAK) as the right Line A B to A D.

For the Chord BK is to the Chord DL, as AB to AD. Alſo as the two Chords BI, IK, to the two Chords DM, ML, ſo is AB to AD. But the two Chords BI, IK, are by Conſtruction equal to the two Chords DG, GE. Wherefore the two Chords DG, GE, are to the two Chords DM, DL, as AB to AD. But as two Chords DG, GE, to two Chords DM, D L, ſo is the whole Arch D E, to the Arch D L; which I thus make appear.

to ML

If the Arches B I and I K, be cut in the Middle in *a* and *b*; likewiſe the Arches D M, M L, being cut in the Middle in *c* and *d*; and the Chords of the Biſegments in the Arch BK being drawn; likewiſe the Chords of the Biſegments in the Arch DL; alſo theſe will be as A B to A D, and they will always be ſo, if the Biſegments of the Biſegments proceed infinitely. Alſo the ſame is true in the Biſections of the Arches KC and LE. But the Chords of the Biſegments infinite in Number, are equal to the Arch itſelf; for if all the Chords were leſs than all their Arches, there might yet a Biſection proceed; which is contrary to the Suppoſition.

Therefore, the Arch DE is to the Arch DL, as the Arch BC to the Arch BK, as the given Line A B, to the given Line AD, and ſo alſo the given Angle BAC to BAK. Therefore the Angle given BAC is cut in K, in the Proportion given of AB to AD. Which was to be done.

DG : GE : DM ML

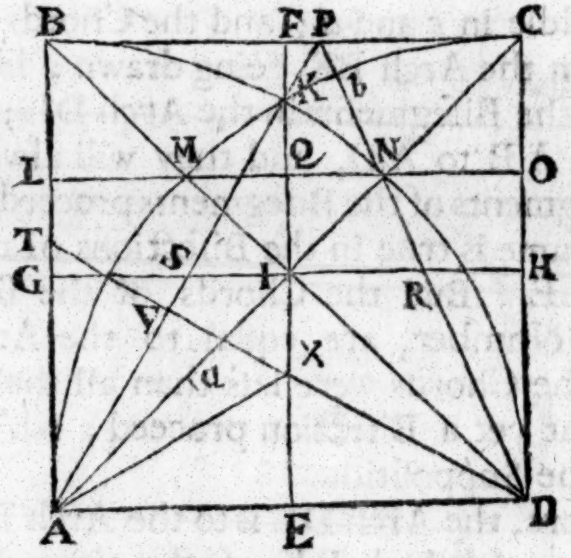
Here we see that ML being left then the Prop is false

PROP. X.

Of Sines, Subtenſes, and other Lines in the Quadrant of a Circle.

THE Tangent of an Arch of 22 Degrees and ½ is equal to the Exceſs whereby the Diagonal of a Quadrate exceeds the Side of the ſame.

Let ABCD be a Quadrate, and in it a Quadrantal Arch inſcribed BD, cutting the Diagonal AC in N. Then AN is equal to the Side AB. I ſay NC is equal to the Tangent of an Arch of 22 Deg. ½.



Deſcribe the Quadrantal Arch AC, cutting the Diagonal DB in M: Alſo draw MN, it will be parallel to the Side BC, and MN being produced to both the Sides AB, DC, in L and O. Then DO will be equal to the Sine of an Arch of 45 Degrees, or to the half Diagonal DI. Let the Quadrate ABCD be cut by the right Lines EF, GH, interſecting at I, four Ways. Then is DO a mean Proportional between the whole Side DC, and his half DH. Wherefore as DC to DO, ſo is DO to DH; and likewiſe the Difference CO to the Difference OH.

Likewiſe CO and NO are equal, becauſe the Angles at C and N are half right Angles. Wherefore NC is in Power double to CO; alſo CO is in Power double to OH (being double in Power ſignifies that a Quadrate whoſe Side is CO, contains as much again as a Quadrate whoſe Side is OH;) for when the Side DC is in Power double to DO, and DC, DO, OH continual Proportionals; CO is in Power double to OH; therefore alſo HO, OC, NC are continual Proportionals; becauſe NC is double in Power to CO.

Draw the Chord DN; then the Angle ODN will be an Angle of 22 Deg. ½. Alſo the Chord DN cuts the right Line GH in R, and HR will be equal to HO.

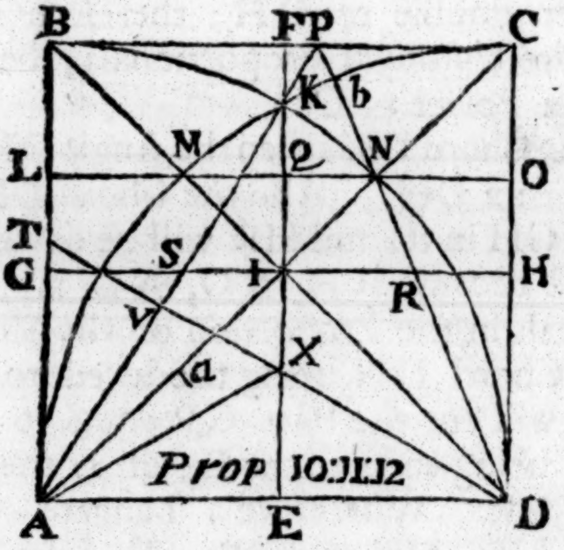
Likewiſe becauſe RH, NO, NC are continual Proportionals in the Proportion of the Side DC to DO, the Chord DN being produced to the Side BC in P, will cut the Part CP equal to the right Line NC, being equal to the Exceſs of the Diagonal above the Side. Wherefore a Tangent of an Arch of 22 Deg. ½, is equal to the Exceſs of the Diagonal, *&c.* which was to be demonſtrated.

Cor. From hence it follows, that the right Line BP is equal to the double of CO. For if from the Centre C, with the Interval CD, an Arch be drawn, cutting CA in *a*, *a*N will be double to IN, and A*a* equal to NC; therefore, when as *a*C, and BC be equal, likewiſe *a*N, that is, the double of CO will be equal to BP.

PROP. XI.

A Tangent of an Arch of 30 Degrees, together with a Tangent of an Arch of 22 Deg. $\frac{1}{2}$, are equal to the Side of a Quadrate BC.

Divide the Arch MC in the Middle in *b*. Then either of the Arches C*b*, *b*M will be an Arch of 22 Deg. $\frac{1}{2}$, and D*b* will paſs by N; alſo MK is a third Part, that is, two Sixths of the Arch CM. Then when as MK is a third Part of MC, and M*b* an half, the Arch MK will be double to the Arch K*b*.



Then the Angle BDK is a ſixth Part of the Angle CDM, that is, a twelfth Part of the right Angle.

Produce D*b* to the Side BC in P.

Then becauſe the Angle AKD is two Thirds, that is eight Twelfths of one right Angle, and the Angle KDP one Twelfth; if AK be produced until it meet D*b* being produced, which will happen in P. For whereſoever it cuts in D*b* being produced, it will make with it an Angle equal to ſeven Twelfths of a right Angle, becauſe the Angle KAB is equal to eight Twelfths, and the Angle KDP to one Twelfth; for the remaining Angle KPD will be ſeven Twelfths of one right Angle; for when the Angle CPD is nine, and the Angle which the Tangent of 30 Deg. makes with his Secant is eight, the remaining Angle will be a Complement to two right Angles, that is, to the three Angles BPA, APD, CPD. Therefore when as the Angle CPD is nine, and the Angle DAK eight Twelfths, the other Angle APD will be ſeven, and all the three Angles together, will be twenty-four Twelfths of one right Angle, that is equal to two right Angles, that is to the three Angles of the Triangle APD. Wherefore the Tangent of 30 Deg. *&c.* which was to be demonſtrated.

ANIMADVERS.

This Prop. hath long ſince been confuted by Tables of Sines, Secants and Tangents, calculated from ſeveral Geometricians by Dr. Wallis.

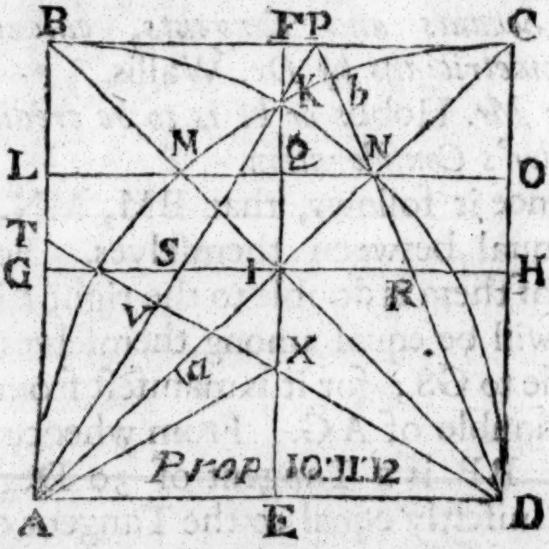
Whether Mr. Hobbs *or he is to be credited, is left to the Reader's Conſideration.*

From hence it follows, that BM, MN, NC, CP, KS, are equal between themſelves. For whereas every one of them is double to the right Line HO or IQ, they will be equal among themſelves. Beſides BP is double to GS; for it is manifeſt from this, that AB is the double of AG. From whence it appears again, that BP is a Tangent of 30 Deg. for AS, which is manifeſtly equal to the Tangent of 30 Deg. is the double of GS.

PROP. XII.

A right Line which cuts the Baſe AK *of an Equilateral Triangle, from any Vertical Point in the Middle, is Seſquialter of the Tangent of an Arch of 30 Degrees.*

LET there be taken in AB, a Tangent of an Arch of 30 Deg. AT. Join DT cutting AK in V, and the right Line EK in X, and draw AX. Then the Triangle A T X will be Equilateral, and both the Angles A V D, and K V D are right Angles; likewiſe the Sides AX, XD will be equal. And TX is the double of VX. Therefore DV is the Triple of VX, that is, Seſquialter (*viz.* once and an half) of DX, that is, a Tangent of 30 Deg. which was to be demonſtrated. The ſame Propoſition is demonſtrated by

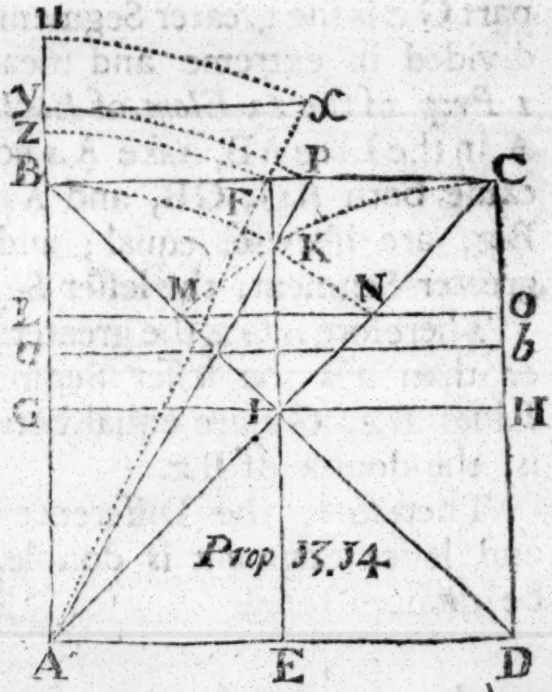


by *Euclid*, in his 18 *Prop.* of the 14 *Elem.* but becauſe it is too long to rehearſe in this Place, let it be read by him that doubts of the Truth of it.

Corol. Therefore the right Line DV or EK, is triple the Difference between the Side DC and the half Diagonal DI or DO; for CO is manifeſted to be equal to half the Tangent AT.

PROP. XIII.

The Difference between the greater and leſſer Segment of a right Line, divided in extreme and mean Proportion, is double the Difference between the ſame right Line, and a right Line whoſe Power is to it, as 5 to 4.



LET the given right Line be AB, to which let there be put at right Angles BF, half of the given Line AB; then draw AF, the Power whereof is to the Power of the given Line, as 5 to 4, and to the Power of BF, as 5 to 1.

NOTE, That the Word *Power* will be often uſed; therefore, by the *Power of a Line* you are to underſtand a Quadrate or Square made from the Line mentioned, each Side whereof is equal to the mentioned Line; as if it had been ſaid, a Qaudrate or Square drawn from AF (that is, a Quadrate, each of its Sides being equal to AF) is to the Quadrate drawn from the given Line AB, as 5 to 4, and to the Quadrate drawn from BF, as 5 to 1.

With the Interval AF, deſcribe an Arch of a Circle, cutting AB being produced in *z*. Then the Square from the right Line A*z*, is to the Square made from the given Line AB, as 5 to 4, and to the Square from BF, as 5 to 1.

I ſay, the Difference between the greater and the leſſer Segment of the given Line AB, divided in extreme and mean Proportion, is the double of B*z*.

Divide AB in the Middle in G; then AG being taken away from the whole Line A*z*, the remaining part G*z* is the greater Segment of the given Line AB divided in extreme and mean Proportion (by the 1 *Prop.* of the 13 *Elem.* of *Euclid.*) From the Point A in the Line AB, take A*a* equal to G*z*; then becauſe both AG, GB, and A*a*, G*z* are equal, G*a*, B*z*, are likewiſe equal; and becauſe A*a* is the greater Segment, the leſſer Segment will be *a*B.

Therefore is G*z* the greater Segment, being greater than *a*B the leſſer Segment. The two right Lines B*z*, G*a* are equal between themſelves, that is, the double of B*z*.

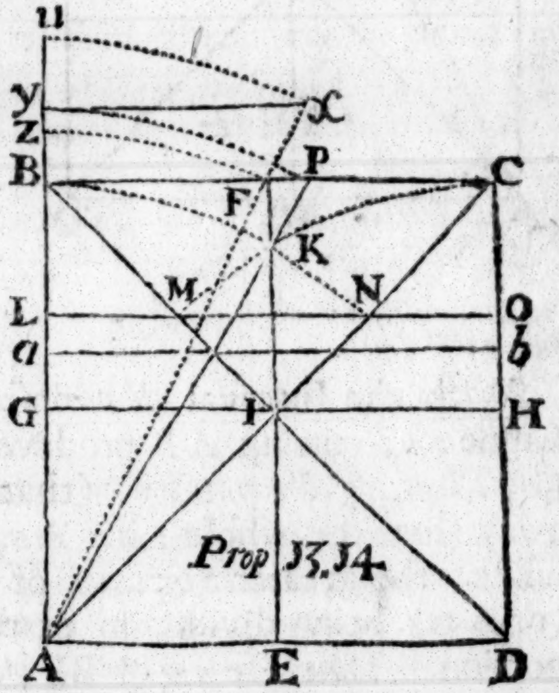
Therefore, the Difference between the greater and leſſer Segment is double, &c. Which was to be demonſtrated.

PROP.

PROP. XIV.

If the Secant of an Arch of 30 Degrees be cut in extreme and mean Proportion, the greater Segment will be equal to the Semidiagonal of a Quadrate made from the Semidiameter.

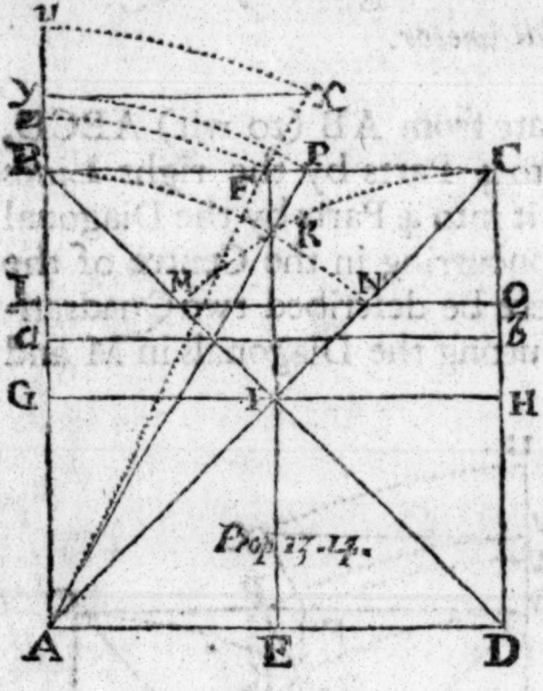
DEſcribe a Quadrate from AB (to wit) ABCD, and divide it into 4 Parts by the right Lines EF, GH, alſo divide it into 4 Parts by the Diagonal Lines AC, BD, all concurring in the Centre of the Quadrate at I; let there be deſcribed two Quadrantal Arches AC, BD, cutting the Diagonals in M and N, and the right Line EF in K draw AK, and produce it to the Side BC in P. Then AP is the Secant of the Arch BK, which is an Arch of 30 Degrees; likewiſe BP is a Tangent of 30 Deg. and the half of the Secant AP.



By the Points M and N, draw the right Line

LO equal and parallel to the Side BC. Wherefore AL, or DO, is equal to the Semidiagonal AI.

I ſay, AL is the greater Segment of the Secant AP divided in extreme and mean Proportion.



With the Interval AP deſcribe an Arch of a Circle P*y*, cutting AB produced in *y*.

From the Point *y* draw *yx* parallel to the Side BC, and equal to half the Secant AP, that is, equal to the Tangent BP. Then A*x* being drawn, is in Power Quintuple to (or five times as much as) the right Line *yx*.

With the Interval A*x* deſcribe an Arch of a Circle *xu*, cutting AB produced in *u*. Wherefore (by *Elem.* 13 *Prop.* 1.) *yx* (that is, BP, being taken away from the whole Line A*u*, the remaining Part will be the greater Segment of the right Line AP (or A*z*) being divided in extreme and mean Proportion. Likewiſe *yx* or BP being taken from A*u*, the remaining Part AL will be equal to the Semidiagonal AI.

For it is manifeſt by *Prop.* 10. that the right Line BP is double to the right Line CO or BL. Therefore *v*L is equal to *yx* or BP; and the remaining Part AL the greater Segment of the Secant AP, or the right Line A*y* divided in extreme and mean Proportion. Which was to be demonſtrated.

Corol. From hence it follows, that the half Diagonal AI is the greater Segment of half the Secant AP, that is, of a Tangent of 30 Degrees, that is, of a Side of a Cube inſcribed in a Circle, whoſe Diameter is AB.

PROP. XV.

A Digreſſion concerning the Diſcord between the Computation of Lines, of Superficies, and of Numbers in the Demonſtrations of Geometricians.

BY the 5th *Definition* of the 5 *Elem.* of *Euclid*, Magnitudes are ſaid to have Reaſon or Proportion to one another, which being multiplied, may exceed one another.

From which Definition it is manifeſt, that Lines, Superficies, and Solids, can have no Proportion among themſelves; for being multiplied together in themſelves, they can never mutually exceed.

If, notwithſtanding, for a Line there be uſed a ſmall right Angle, there may truly ſometimes a Proportion be found between a Superficies and that right Angle, tho' it be very ſmall; to wit, when two Quadrates are to each other as a Quadrate Number to a Quadrate Number. And one is the Meaſure of the other, becauſe they may be compared.

But the Quadrates which are between themſelves as Quadrate Numbers, are much fewer than thoſe which are not as Quadrate Numbers, altho' both are innumerable.

Therefore the Doctrine concerning the Quantity of Lines is a Science ſubſiſtent by itſelf, and diſtinct from the Science of Superficies, and this diſtinct from the Science of Solids.

Moreover, becauſe all Menſuration begins from a Point, and a Point cannot be conſidered as Figurative, how can a Point in a Quadrate Angle any otherwiſe be conſidered, than as a *Quantum* common to the Quadrate and Side of it? For to conſider it as Figurative, and not Figurative, is abſurd.

Beſides, how can a Point, which is in the Centre of a Circle, be accounted for nothing, when as it is diviſible? For in how many Parts ſoever any Sector is divided, in ſo many Parts likewiſe is the Centre divided.

Perhaps, ſome may ſay, without the known Quantity of Figures, very few of Theorems hereafter concerning the Proportions of Lines are demonſtrable, beſides Mechanick Menſuration. But they err, Firſt, Becauſe the Proportions of Lines between themſelves, and the Conſtruction of Figures, and all their Affections of Motion, are deliver'd by *Euclid*, without the known Quantity of a Quadrate, or of any other Figure, or the Proportion of Figure to Figure. Neither hath any one attempted to demonſtrate the Length of a Line by the Magnitude of a Quadrate, before *Archimedes*; neither after him (that I know) beſides *Eutocius*, before *Copernicus*; truly neither hath he demonſtrated accurately.

Neither do I ſay theſe things, becauſe I would admit Mechanick Operations for lawful Demonſtrations. But in every Geometrical Queſtion, I think it much more Prudence, before mechanically Meaſuring the Magnitude ſought, as much as may be done with Truth, to attempt the neareſt Way, and afterwards inquire into the Cauſe of that nearneſs, which being found out, will detect the Truth or Falſhood; than raſhly believing of uncertain Reaſoning, or the Reckoning of thoſe, or the Authority of others, who pronounce unknown things, eſpecially, not only where the Certainty of one, but of many Theorems is deſtroyed. A diligent Meaſurer declares more credible things by Meaſuring, than he that reaſons from falſe Principles; and will deſervingly ſcorn the Algebraiſts, that is, the Arithmeticians, diſputing againſt Meaſure, and ſaying, That the Side of a Quadrate, and the Root of a Number are the ſame.

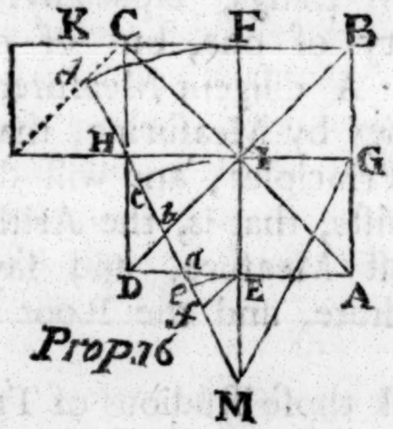
Alſo no Man will think thoſe ſtudious of Truth, who ſeeing a Concluſion contrary to their own Knowledge, although ſupported with very probable Arguments, are content to reſiſt the ſole Demonſtration, and neglect the Verity of the thing (which ſometimes proceeds from the Debility of their own Wit, or from the Omiſſion of ſome Propoſition, which the Demonſtrator ſuppoſed to be well known to Geometricians, eſpecially the chiefeſt of them) for theſe Kind of Men are not ſeeking the Truth, but Victory.

What I have ſaid concerning the Incongruity of Lines and Superficies, will appear clearly in the following Problem, being propoſed to Algebraiſts.

PROP. XVI.

DEſcribe the Quadrate ABCD, and divide it (both) with the right Lines EF, GH, and with the Diagonals AC, BD, interſecting in I four Ways.

In FC being produced, take FK equal to the half of MF, and join MK; then with the Radius MF, deſcribe an Arch F*d*, it will cut in *d*, ſo that MF, M*d* are equal. And MK cuts the right Line E*d* in *a*, and the Diagonal ID in *b*. Alſo MK will paſs by H.



Prop.16

Draw E*f* parallel to ID cutting MK in *f*. Moreover with the Radii ME, MI, deſcribe the Arches E*e*, I*c*, cutting MK in *e* and *c*.

It is manifeſt from this Conſtruction; Firſt, That M*a*, *a*H, HK, are Equals between themſelves.

Secondly, (Becauſe MI is double to ME) M*b* is double to M*f*.

Thirdly, (Becauſe ME is the double of E*a*) M*f* is the double of *fa*; and (becauſe MI is double to IH) H*b* is double to *ba*; and then it will be as H*a* to H*b*, ſo M*a* to M*f*, as 3 to 2, or 9 to 6.

Fourthly, It is manifeſt, that as well K*d* as *ba* is to *ae*, as 3 to 1, and to H*c* as 3 to 2, and ſo likewiſe *fe* will be to *ae*; to wit, as 3 to 2, or 9 to 6, and therefore C*d* being joined, is parallel to I D; therefore *d*H, H*b* are equal.

Fifthly, It is manifeſt, that M*f*, *fb* are equal, and *b* K, *d a* are equal.

Then M*d* is four times two, whereof MK is thrice three.

Wherefore MK is to M*d* or MF, as 9 to 8.

Then becauſe MK is Quintuple in Power to FK or M*b*; if FK be taken away from the right Line MK, the remaining Part *b*K (by *Elem.* 13. *Prop.* 1.) will be the greater Segment of MK, or *bd* of M*d* being divided in extreme and mean Proportion.

But *b*K is equal to *da*. Wherefore *da* is the greater Segment of M*d* divided in extreme and mean Proportion.

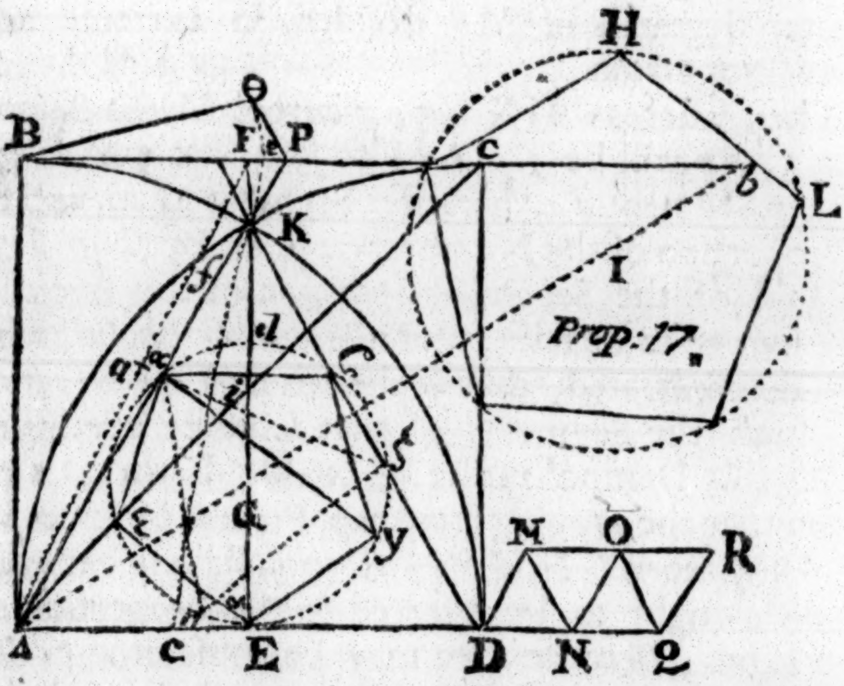
Then whereas MK is 9, whereof M*d* is 8, and M*a* 3, *da* will be 5. And ſo the whole M*d*, his greater Segment *da*, his leſſer Segment M*a*, will be in Proportion of the Numbers 8, 5, and 3; which at the End of the Second Propoſition of this Book, I have ſhewed to be falſe; and it is againſt *Euclid*, who hath demonſtrated, that if the whole Line be rational, both the Segments of that Line be irrational. Unleſs this Demonſtration be confuted, there is no reaſon that the arguing from the Powers of Lines be heard any more. No Line drawn obliquely between Parallels ought to be referred to the Proportion of pure Lines; becauſe they may be conſidered either as Triangles, or as ſmall oblique angled Parallelograms, whoſe Longitude is not determined. For the Longitude of a Figure is nothing ſurely beſides that which is called Altitude.

PROP.

PROP. XVII.

The Side of an Icoſahedron is equal to the third Part of the greateſt Semicircle in its own Sphere.

DEſcribe the Quadrate ABCD, and divide it in the Middle by the right Line EF parallel to the Sides AB, DC; alſo deſcribe a Quadrant ABD, whoſe Arch will cut the right Line EF in K. Join AK



being produced to BC in P. Then is BP a Tangent of 30 Degrees, to which add P*b* equal to the Side BC, the whole Line B*b* will be compounded of the Side, and a Tangent of 30 Deg. Likewiſe draw the

Diagonal A C, which the right Line E F will cut in the Middle in *i*.

Join D K, and A K D will be an Equilateral Triangle.

In the Side A D take A *c* a 3d Part of it, and join *c* K, and produce it to the Side B *c* in *e*; and F *e* will be a 3d Part of F P, and therefore B *e* will be a 3d Part of the whole Line B *b* (by *Conſect.* 3. of the 7 of this.)

In the right Lines K A, K D, let there be taken K α, K β, either of them equal to a 4th Part of the Diagonal A C, or the Half of A *i*, and join $\alpha\beta$. Then the Triangle K $\alpha\beta$ will be Equilateral; and its Baſe $\alpha\beta$ will be cut by the right Line K E in the Middle, and at right Angles.

Of the right Line B P, which is a Tangent of 30 Degrees, and of 2 right Lines, whereof either is equal to $\alpha\beta$, make the Triangle $\alpha\beta\gamma$; and $\beta\gamma$, $\alpha\beta$, will be equal.

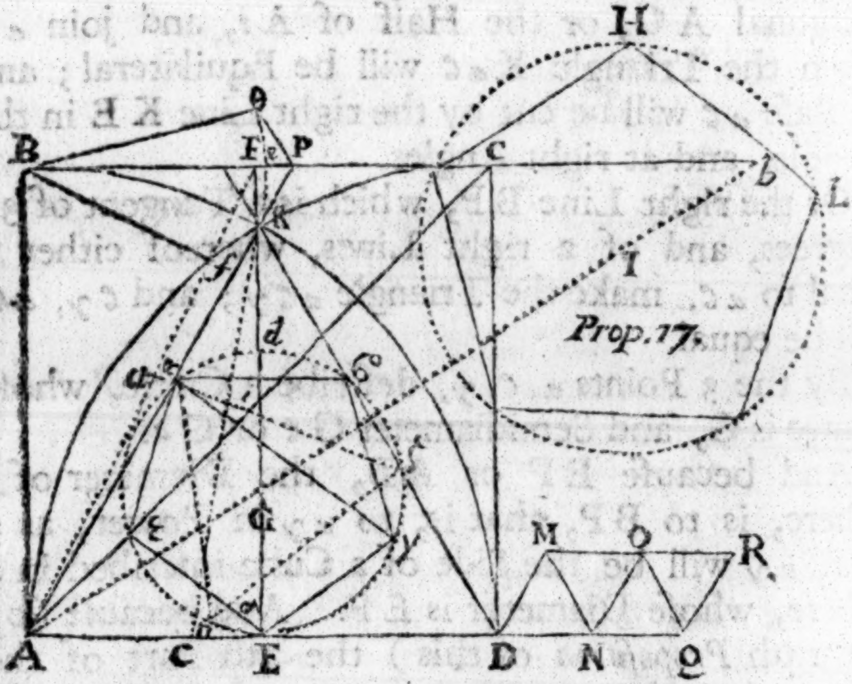
By the 3 Points α, β, γ, deſcribe a Circle, whoſe Centre is G, and Semidiameter G β or G α.

And becauſe E F or A B, the Diameter of a Sphere, is to B P, that is, to $\alpha\gamma$ in Power, as 3 to 1, $\alpha\gamma$ will be the Side of a Cube inſcribed in a Sphere, whoſe Diameter is E F. And becauſe (by the 14th *Propoſition* of this) the 4th Part of the Diagonal, is the greater Segment of the Side of a Cube, divided in extream and mean Proportion; it will be (by the 8 *Prop.* of the 13 *Elem.*) that the right Line $\alpha\beta$ is the Side of a Pentagon in the Circle $\alpha\beta\gamma$; and that Side of a Pentagon one of the 12 Seats or Points of a Dodecahedron inſcribed in the ſame Sphere with the Icoſahedron.

Compleat the Pentagon $\alpha\beta\gamma\delta\varepsilon$.

Draw the right Line A F cutting the Arch A K C in f; and the Power of B f will be (as is well known to Geometricians) a 5th Part of the Power of the Side A B, or Quintuple in Power to the Diameter of the Sphere.

Deſcribe aloof off with the Radius I H, being equal to B f, the Circle H L, in which the Side of an Equilateral Pentagon is H L. Then H L will be the Side of an Icoſahedron in the ſame Sphere, by the 16 *Prop.* of the 13 *Elem.*



Then (by the 5 *Prop.* of the 14 *Elem.* of *Clavius*'s Edition) the right Line H L is the Side of an Equilateral Triangle inſcribed in the ſame Circle. Inſcribe in the Circle $\alpha\,ϛ\,\gamma\,\delta\,\varepsilon$, the Equilateral Triangle $\alpha\,\zeta\,\eta$. Then $\alpha\,\zeta$ will be the Side of an Icoſahedron in the Sphere, whoſe Diameter is E F.

It is to be ſhewn, that the right Line $\alpha\zeta$ is equal to a 3d Part of the Semicircle, whoſe Semidiameter is BF, that is, to the Arch BK, which is a 3d Part of a Quadrant of a Circle deſcribed from AB as a Semidiameter.

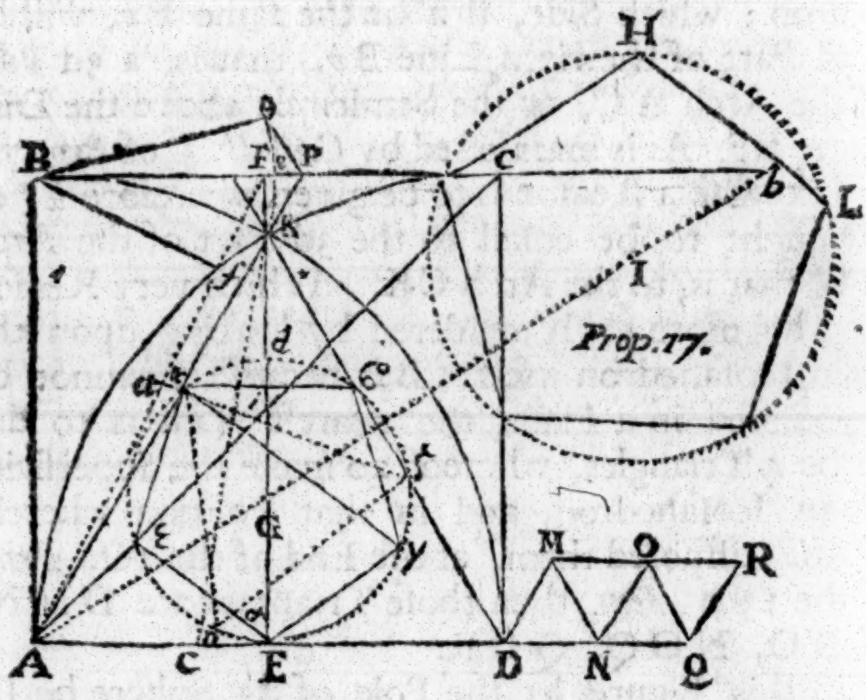
With the Semidiameter B*e*, deſcribe an Arch $e\theta$, in which take $e\theta$ equal to the Arch $\gamma\zeta$. Then draw $B\theta$, it will be equal to $\alpha\zeta$, that is, to the Side of an Icoſahedron in a Sphere whoſe Diameter is EF. Then with the Radius $B\theta$, deſcribe an Arch of a Circle cutting BP, it will give a Side of an Icoſahedron; which Side, if it be the ſame B*e*, will be a 3d Part of the right Line B*b*, that is, a 3d Part of the Arch AC, or the Semicircle above the Diameter EF. As is manifeſted by *Conſect.* 3. of *Prop.* 7.

Likewiſe a Reaſon is to be given wherefore $\alpha\zeta$ or $B\theta$ ought to be equal to the 3d Part of the Arch AC, that is, to the Arch CK. Which very Reaſon may be more eaſily rendered by looking upon the Solid Icoſahedron itſelf. But becauſe it cannot be expounded in a Plain, the neareſt Way is to deſcribe 4 Triangles, whereof 20 make the Superficies of an Icoſahedron, and in that Poſition wherein *Clavius* diſpoſed them, at the End of the 16th *Prop.* of the 13th *Elem.* then thoſe Triangles are DMN, MNO, NOQ, QOR.

In this Figure let the Pole of the Sphere be D, then the Points D, M, N, O, Q, R, will be in the Concave Superficies of the Sphere; and therefore the right Lines MN, NO, OQ, will not be in the ſame Plain with the Points D and R, which are in the Plain by the Diameter of the Sphere.

Wherefore the Side of an Icoſahedron proceeds from the Pole D to the Pole R, by 5 equal right

Lines, to wit, from D to M, from M to N, from N to O, from O to Q, from Q to R. And firſt in the Motion from D to M, it is moved forward towards R. Again, from M to N, it is not moved forward towards R. Thirdly, from N to O, it is moved forward, as much as from D to M. Fourthly, from O to Q it is not moved forward towards R. Fifthly, from Q it is moved forward to the very Point R. Therefore by 5 equal right Lines, which are the Sides of an Icoſahedron, is the Motion made



from Pole to Pole; but by reaſon of the Digreſſion to the neareſt Circles, which divide the Superficies of the Sphere in 5 Parts, the Motion is made by the 5 Sides of an Icoſahedron.

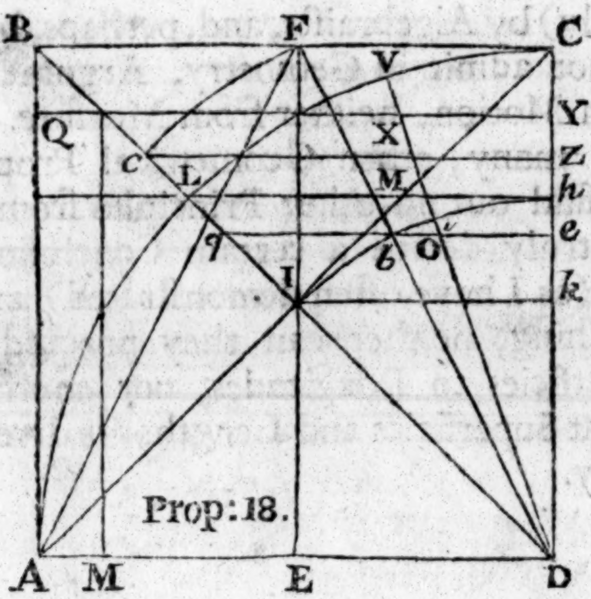
But in the Circumference of a Semicircle, the Motion is made from Pole to Pole by 3 Arches, every one of them being equal to the Arch CK or BK. The Motion therefore being taken away, which is made by thoſe two Sides of an Icoſahedron which move not forward, the Ways by the 3 Sides of an Icoſahedron, and by 3 Arches of a Semicircle, every one of which being equal to the Arch CK, or at leaſt, by the Chords of thoſe Arches, will be equal. But B θ will be found greater than the Chord BK. Wherefore the Arches every one being equal to BK, are equal to 3 Sides of an Icoſahedron, and one to one. Which was to be demonſtrated.

This kind of Demonſtration will be condemned (I know ſurely) by Algebraiſts, and, perhaps, by others, who do not admit in Geometry, Arguments to be taken from Motion, neither from Meaſure. But to this, and many other Geometrical Propoſitions, they can find out no other Principles from whence they can truly derive a certain Concluſion. For Numbers (as I have often demonſtrated) are unapt for this Thing, neither can they proceed rightly from Superficies to Longitudes, nor contrarywiſe, becauſe that Superficies and Lengths are divers kinds of Quantity.

PROP. XVIII.

A Circle being given, to find a Quadrat equal to it.

LET the Circle given, whoſe Quadrat is D A C, be inſcribed within a Quadrat A B C D; and D L C the 8th Part of the Circle. Cut the Quadrat A B C D with the Diagonals A C, B D; alſo with the right Lines E F, I *k*, all interſecting in I, and dividing the Quadrat into 4 Parts both Ways; and draw D F cutting the Arch C L in P, and by the Point P draw Y Q, cutting the Diagonal B D in Q; then D Y, Y Q will be equal.



With the Radius D F, deſcribe an Arch *Fc* cutting the Diagonal B D in *c*, ſo that D F, D *c* may be Equals.

I ſay, the Quadrate from YQ or DY is equal to an 8th Part of the Superficies of the Circle DCL.

From the Point *c* draw *cz* parallel to YQ, cutting DC in *z*. Then is *zc* (by the 6th Prop. of this) half the Arch AC, and therefore equal to the Arch CL.

In the Arch CL, take the Arch LV equal to CP, and join DV, cutting YP in X; and the 3 Lines CYP, will be wholly within the Sector DCL; but the 3 Lines PQL are all without the ſame Sector.

Likewiſe both the 3 Lines together are (as before I have ſhewn, and now will ſhew) equal to the Sector APV. For becauſe the right Line BC is cut in the Middle at F, and the Baſes of the Triangles DCB, DYQ are parallel; alſo the Baſis YQ is cut in the Middle in P, and the Triangles DYP, DPQ are equal.

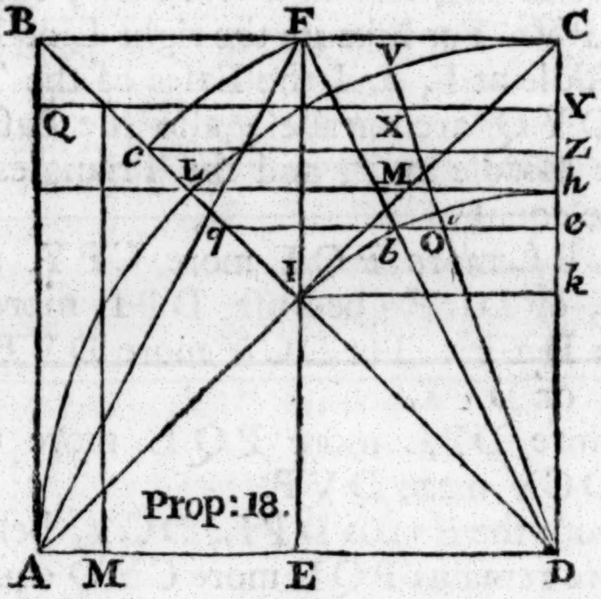
Now DPL more PQL more CPY, are equal to DVL, or DCP (becauſe DPL more PQL is equal to DYP). For DCV more DVP is equal to DCP, or DVL.

Wherefore DPL more PQL more CYP is equal to DCV more DVP.

Then both the Equals DPL, DCV, being taken away, there remains PQL more CYQ equal to the Sector DVP. And hitherto our Adverſaries conſent or agree. This alſo they muſt condeſcend to (for it is manifeſt) that if CYP, PQL, are equal between themſelves, the Triangle DYQ, and the Sector DCL are likewiſe equal.

And I had thought it demonſtrated before from this, becauſe the Triangle whoſe Vertex is D, and Baſis parallel to the Side BC, no Triangle can be made equal to the Sector DCL, whoſe Baſe paſſeth not by P; but the Force of the Demonſtration they have not clearly underſtood. Go to then, we will produce, if it be poſſible, a more clear Demonſtration.

With the Radius DI, which is a mean Proportional between the Side DC, and the half Side D*k*, deſcribe an Arch of a Circle cutting DF in *b*, and DV in *i*, and DC in *b*; and by the Point *b* draw *eq*, cutting DC in *e*, and the Diagonal BD in *q*, and DV in *o*. Then the Sector DCP will be dou-



ble of the Sector D*bb*, as alſo of the 4 Lines CP*bb*; likewiſe the Triangle DXY double of the Triangle D*oe*; it is likewiſe double of the 4 Lines YX*oe*. Wherefore the remaining Sector DVP is double to the 3 Lines CYP, and likewiſe double to the 4 Lines VP*bi*. And D*bi* double to the 3 Lines *bbe*.

Then when as the Sector D V P is double to the three Lines C Y P, and the ſame equal to two three Lines, C Y P, P Q L; then C Y P, P Q L will be equal between themſelves.

Which was to be demonſtrated.

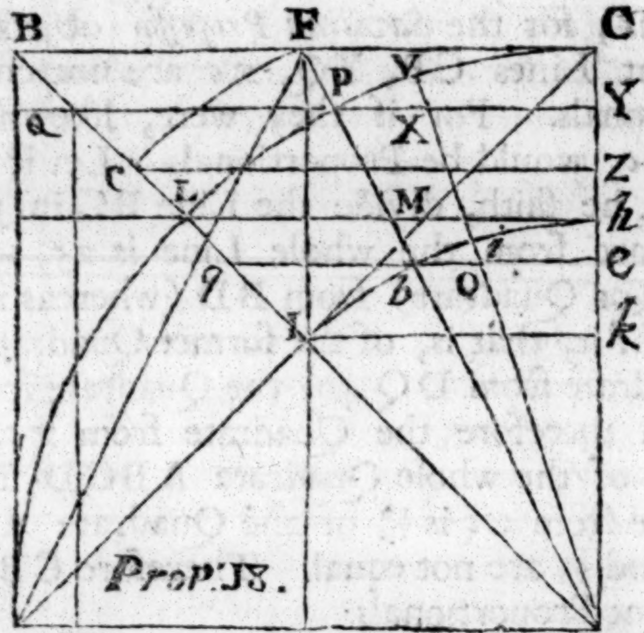
Therefore a Quadrate is found (to wit, a Quadrate from Y Q) equal to the 8th Part of a Circle, to wit, to the Sector D C L; and ſo is effected the ſquaring of a Circle, nor now the firſt Time, but many Years ago ſufficiently demonſtrated by divers Methods.

From this Demonſtration likewiſe I have deduced the duplicating of a Cube, ſhewing that the 4 right Lines CB, *zc*, *eq*, and I*k*, are continual Proportionals. But neither have the Algebraiſts underſtood this; for the *Savilian Profeſſor* objects, that the right Lines C B, Y Q, *zc* are not continual Proportionals. For if they were, likewiſe B D, D Q, D*c*, would be Proportionals. Let it be ſuppoſed as he ſaith, divide the Line B C in 5 Parts, the Square from the whole Line is 25, and the Square (or Quadrate) from B D (whereas it is the Double of it, that is, of the former Quadrate) 50; the Quadrate from D Q 40, the Quadrate from D*c* 32; and therefore the Quadrate from *zc*, 16 of thoſe 25 of the whole Quadrate A B C D. But the Quadrate from *zc* is $\frac{10}{16}$ of the Quadrate A B C D. But $\frac{16}{25}$ and $\frac{10}{16}$ are not equal. Therefore C B, Y Q, *zc* are not Proportionals.

But Arguments of this Kind (for the Cauſe declared at the 15th Propoſition) are the mere Spirit of Deluſion in Algebraiſts, in applying thoſe Numbers to Quantities, which have not the Proportion of Number to Number.

If BC be divided in 5 Parts, the Quadrate from DF will be $6\frac{1}{4}$, to wit, the 4th Part of 25. Wherefore D*c* will be $31\frac{1}{4}$, and the Quadrate from BD will be 8 Times as much as the Quadrate from BF, or 50.

But the mean Proportional between 8 and 5, will be the Side (I call it the Side, not the Root) of the Number 40. Then not only 50, 40, 32, but alſo 50, 40, $31\frac{1}{4}$ will be continual Proportionals. Therefore what the Cauſe of this Diſcord ſhould be (ſince it is plainly demonſtrated, that both *zc* is equal to Half the Arch AC, and that the Quadrate from YQ is equal to the Sector CL) unleſs that Lines drawn (that is diviſible according to Breadth) cannot be compared with pure Lines, that is, without Breadth. But the right Lines CB, *zc*, *eq*, *k*I, are continual Proportionals, as I ſhall make appear moſt clearly in the following Demonſtration.



CONSECT. I.

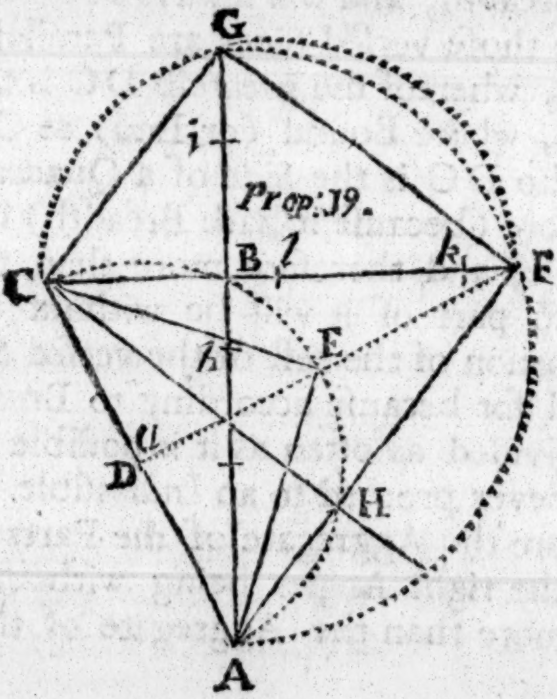
The Arch CL is leſs than four fifths of the Radius DC; for all the verſed Sines which are poſſible to be drawn in the Quadrant DAC, being taken together, are equal to the Area or Content of the Quadrant, and by conſequence to the Quadrat DYQM, or to four fifths of the Quadrate ABCD. And for becauſe thoſe verſed Sines are all bounded within the Arch, the 2 Sides BC, CD of the Quadrat ABCD, ought to be to the Halfs of the Arch AC, in Proportion double of CD to DY; but it is not ſo. For altho' you ſhould divide Lines, or other continued Quantity infinitely, notwithſtanding you will never attain to nothing; becauſe continual Quantity is always diviſible into Diviſibles. Then thoſe Sines, whereof the whole Number of them fill the Area of a Quadrant, will have every one their Breadth, and will always be finite in Number. Alſo thoſe verſed Sines are Parallels between themſelves, whereof the greateſt DC is the Radius of a Circle, whoſe Bound (or End) at C is a ſmall Arch. Alſo DC is the Side of a Quadrat ABCD, and therefore (becauſe it hath Breadth) it will be a right Angle, and therefore more than the Radius DC. And part of it will be without the Circle. The Proportion of the reſt of the verſed Sines is the ſame; and for becauſe according to Breadth, they may be divided as often as it is poſſible to divide, they will never proceed to an Indiviſible.

Therefore the Aggregate of the Parts compaſſed about of the right Angles being without the Quadrant, is more than the Aggregate of the Parts of the Quadrant itſelf AC. Alſo the Arch AC is ſome ſmall matter leſs than four fifths of the Diameter, and the Arch CL, or the right Line *zc* leſs than four fifths of the Side DC.

PROP. XIX.

Between a right Line given, and the Half of it; To find two mean Proportionals.

LET the given Line AB, and the Half of it BC, be diſpoſed at right Angles; join AC, and let AC be cut in the Middle in D; from the Centre D with the Radius DA, deſcribe a Semicircle, and divide it in the Middle in E. Draw AE, CE, and in CB being produced, put BF equal to CE. In AB being produced, take 2 5ths of the right Line AB, which let be B*i*, and between AB and B*i* find



find a mean Proportional B*k*: Equal to it in AB produc'd, put BG (which, as above, is demonſtrated, is equal to 2 5ths of the Quadrantal Arch deſcribed from AB) and draw FG, GC. Moreover, divide CF in the Middle in *l*, from the Centre *l* with the Radius *l*F, deſcribe a Semicircle by F and G, which will paſs by C; which I ſhew thus.

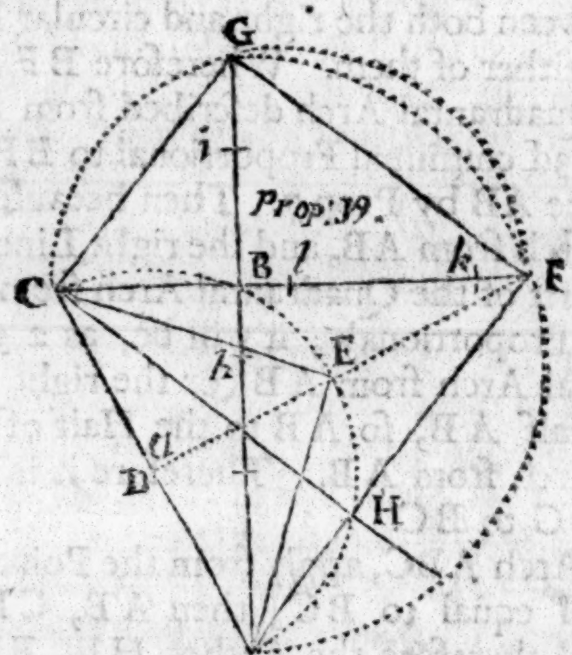
The right Line AC, whoſe Quadrate is Quintuple of the Quadrate BC, is a mean Proportional between the Quadrantal Arch deſcribed from AB and its Half, as is ſhewn at Prop. 6. Therefore CE is equal to half the Quadrantal Arch deſcribed from AB, for becauſe the Angle by E is a right Angle, CE is a mean between both the right and circular Lines CE, EA, and either of them. Wherefore BF is equal to half the Quadrantal Arch deſcribed from AB. And BG is a 3d continual Proportional to BF, and the whole Line AB by Prop. 7. Then becauſe the Quadrantal Arch from AB, and the right Line AB, and 2 5th Parts of the Quadrantal Arch from AB, are continual Proportionals; it will be, as 2 5ths of the Quadrantal Arch from AB (or the right Line B*k*) to the Half AB, ſo AB to the Half of the Quadrantal Arch from AB. Therefore it is as AB to BF, ſo BG to BC.

In the Arch ABC, apply from the Point A, a right Line AH equal to BC; then AB, CH will be equal, and therefore the Arches HE, EB, or the Angles HCE, ECF equal, and the right Lines HF, BF, likewiſe CF, AF are equal, and DE produc'd, divides the Angle AFC in the Middle.

Alſo the Triangles ABF, GBC are alike, wherefore the Angles BGC, BAF are equal, alſo the Angles BCG, BFA equal. And (for becauſe the Angle CHA is a right Angle, and CH equal to AB) the Triangle ABF, FBG are alike and equal. Therefore both BF, GC, alſo CH (or AB) and FG, are equal between themſelves.

And AG being divided in the Middle in *b*, from the Centre *b*, with the Radius *b* A, a Semicircle being deſcribed, it will paſs by F.

Then is FGCH right Angled, and the Triangles ABF, FBG alike. Therefore it is as AB to FB, ſo FB to BG, and ſo BG to BC. Therefore BF, BG are found to be two mean Proportionals between the given Line AB, and the Half of it BC. Which was to be done.



C o r.

It is manifeſt from the Precedings, that the greater of the 2 Means is an 8th Part, and the leſſer Mean

2 5th Parts of the Quadrantal Arch deſcribed from A B. Alſo A B (or C H) is equal to F G; and B G is equal to that Part of the right Line C H, which is cut off by A B, being computed from the Point C, and the Conſtruction of the Problem demonſtrated to the Eyes of all Men.

PROP. XX.

Of the Centre of Gravity of a Quadrant of a Circle.

THE Centre of Gravity (that is, the Centre of Weightineſs or Heavineſs) of a Quadrant of a Circle is in a right Line from its Centre, dividing the Arch in the Middle, and diſtant from the Centre of the Circle, ſo much as is the mean Proportional between the Semidiameter, and 2 5th Parts of it.

Deſcribe the Quadrate A B C D, and in it a Quadrant A D C, and draw the Diagonals A C, B D, whereof B D will cut the Arch A C in the Middle in L.

Between the Semidiameter D C, and 2 5th Parts of it, find a mean Proportional D R, and with the Radius D R, deſcribe an Arch of a Quadrant R S, cutting the Diagonal B D in *z*. I ſay *z* is the Centre of Gravity of the Quadrant D A C.

Cut the Quadrate A B C D into 4 Parts by the right Lines E F, G H, cutting themſelves mutually and at right Angles in I. Join D F, cutting the Arch A C in P; and by P draw Y Q parallel to B C, cutting the Diagonal B D in Q, and E F in V; alſo compleat the Quadrate D Y Q M, whoſe Side Q M cuts the Arch A C in N.

Then as the Quadrate from D F, to the Quadrate from DP or DC, ſo is the Quadrate from DC to the Quadrate from DY. And the Quadrate ABCD is 5, whereof the Quadrate DYQM is 4.

Join YM, and it divides DQ in the Middle; alſo draw the right Lines *Pk*, NO, that parallel to the Side AB, this parallel to the Side BC, they will cut each other mutually, and at right Angles, in the Middle of the right Line DQ.

Therefore becauſe the Quadrate ABCD is to the Quadrate DYQM as 5 to 4, the Quadrate of DQ will be 8, whereof the Quadrate from YQ is 4, and the Quadrate from DC 5, and the Quadrate from the half DQ 2.

Then the Quadrate from the Half of DQ, is 2 5ths of the Quadrate ABCD; therefore the Half of DQ is a mean Proportional between the Semidiameter DC, and 2 5ths of it, and therefore equal to D*z*. Likewiſe *z* is both the Centre of Magnitude, and alſo the Centre of Gravity of the Quadrate DYQM; and the Point I the Centre of Magnitude and of Gravity of the Quadrate ABCD.

Alſo it is ſhewn at the 18 Prop. of this, that the Quadrate DYQM, and the Quadrant DAC between themſelves are equal. And that the 3 Lines CYP, PQL, AMN, NQL between themſelves are equal. Therefore if from the Quadrate DYQM be taken two three Lines equal to **PQL**, **NQL**, and to the ſame Quadrate be added two three Lines CYP, AMN at equal Diſtances from the Diameters of even Height, the 3 Lines CYP, AMN will be equiponderant (or weigh alike) but the Diſtances YP, MN, PQ, QN are equal and alike diſtant from the Diameters of equal Height NO, *Pk*; likewiſe the Points Y, Q, M, D are equally diſtant from the

Centre of Gravity of the whole *z*; therefore *z* is the Centre of Gravity of the Quadrant D A C, which was to be demonſtrated.

CONSECT.

The Centre of Gravity of a Semicircle, is a mean Proportional between two Fifths and one Fifth of the Arch A C. For it is ſhewn (*Prop.* 5.) that the right Line, which is a mean between the Semidiameter, and two Fifths of it, is equal to two Fifths of the Arch A C; therefore if *z* O be produced to *p*, ſo that *z* O, O *p* be equal, then draw *D p*, *p* will be the Centre of Gravity of a Quadrant equal to D A C: And becauſe the Point O divides *z p* in the Middle, the Point O will be the Centre of Gravity of the double of the Quadrant D A C, that is, of a Semicirle deſcribed with the Radius DC; alſo *D p* produced, cuts the Arch C *l* equal to the Arch L C. Then when as the Quadrant D A C reſteth in *z*, and the Double of the Sector C *l* reſteth in *p*, the whole Semicircle will reſt in O: But *z* O is a mean Proportional between D *z*, and its Half, that is, between two Fifths, and one Fifth of the Arch A C, becauſe the Triangle DO *z*, is right angled and equicrural; (*viz.* it hath two Sides equal.)

PROP. XXI.

The Centre of Gravity of the Circular Line and Strait Line, ALCA *is in the Diagonal* BD, *diſtant from the Point* B, *ſo much as is the Length of a Tangent of* 30 *Degrees.*

FInd a Tangent of 30 Deg. B X, equal to which, from the Point B in the Diagonal BD take B T. I ſay that the Point T is the Centre of Gravity of the two Lines A L C A.

For becauſe the Quadrat from A B, is to the Quadrat from B X, or B T, as 3 to 1, and the Quadrat from the Semidiagonal B I is half of the Quadrat from A B, the Quadrat from B I will be to the Quadrat from B T, as $\frac{3}{2}$ to 1, as 3 to 2.

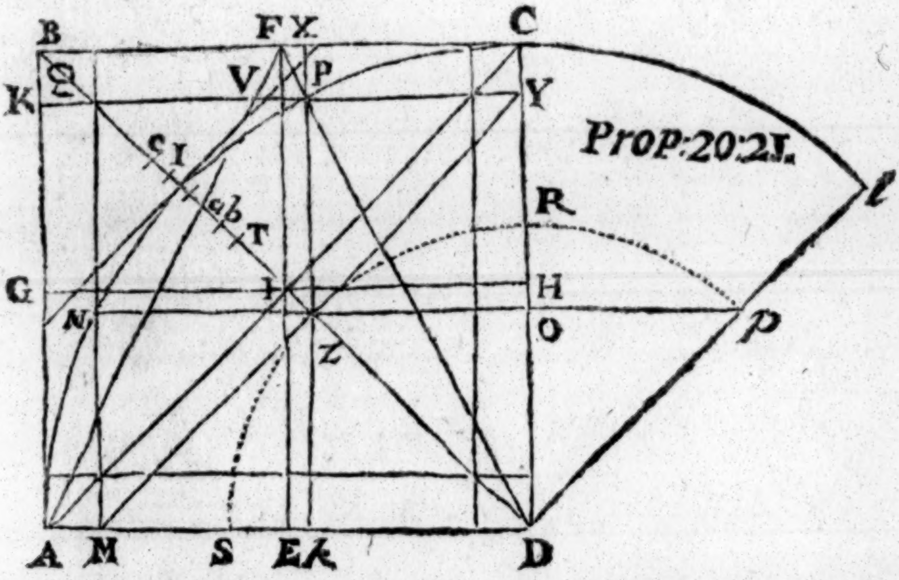
And the three Lines A B C L A, to the two Lines A L C A, as 2 to 3: Which I thus ſhew.

The Gnomon Y B M, is a fifth Part of the Quadrat D Y Q M, that is a fifth Part of the Quadrant D A C: Wherefore alſo the three Lines A B C L A is a fifth Part, or two tenth Parts of the Quadrat A B C D; and the Triangle A B C is the Half, or five Tenths of the Quadrat A B C D: But the Quadrant D A C is four Fifths, or eight Tenths of the Quadrat A B C D; therefore the two Lines A L C A is three Tenths of the Quadrat A B C D.

Then is the Proportion of the three Lines to the two Lines, the ſame which 2 is to 3, that is, reciprocal of the Proportion, as well of the Magnitudes ALCA, ABCLA, as alſo of the Quadrats BI, BT. Join A F cutting the Diagonal B D in *a*. Then becauſe A B is double to B F, and the Angle A B F divided in the Middle by the right Line B *a*, A *a* will be the double of *a* F: Therefore the Point *a* is the Centre of Gravity of the whole Triangle A B C.

Cut *a* T in the Middle in *b*, and take *a c* the Triple of T *b*, and let *a* be the Centre of the Ballance. It will be therefore as the three Lines A B C L A, to the two Lines A L C A, that is, as 2 to 3, ſo reciprocally *c a*, to *a* T, to wit, as 3 to 2.

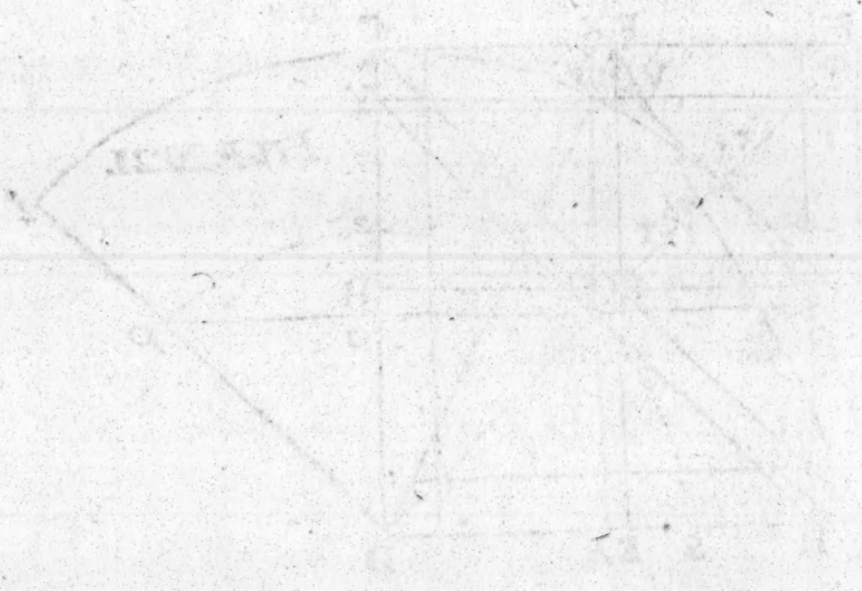
Therefore is T the Centre of Gravity of the two Lines A L C A; and the Point *c*, the Centre of Gravity of the three Lines A B C L A: Which was to be demonſtrated.



The End of the SECOND BOOK.

[illegible] rated in the Middle by the right Line B e, A [illegible] be the double of [illegible] E; Therefore the Point g is the Centre of Gravity of the whole Triangle B C.

[illegible] a T in the Middle in b, and take e c the T [illegible] b, and let c be the Centre of the Balance, [illegible] will be therefore as the three Lines A B C L A, [illegible] two Lines A L C A, that [illegible] to 3, to [illegible] e c, to [illegible] to wit, as [illegible] Therefore T the Centre of Gravity of the two Lines A L C A, and the Point c the Centre of Gravity of the three Lines A B C L A: Which was [illegible] demonstrated.



SOME

PRINCIPLES

AND

PROBLEMS

IN

GEOMETRY,

Thought formerly Deſperate;

NOW

Briefly Explained and Demonſtrated.

The THIRD BOOK.

Written in *Latin*

By *THOMAS HOBBES*:

And made *Engliſh*

By *VEN. MANDEY.*

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SOME
Principles *and* Problems
IN
GEOMETRY, *&c.*

CHAP. I.

Of the Subject, Principles, and Method of the Mathematicks.

HE that would apply himſelf to the Study of the Mathematicks, muſt, in the firſt place, underſtand what the Matter is concerning which Mathematicians treat, and what it is that is enquired into about that Matter.

We are therefore to know, that the Matter which they treat of, is every thing that hath Magnitude, that is to ſay, all about which the Queſtion may be asked, *How great is it?* So that the ſubject Matter of Mathematical Sciences, are the *Lengths*, *Superficies* and *Thickneſs*, or Profundity of Bodies: For every Body hath three, and no more Dimenſions, and theſe differing in Kind. From the Motion of a Body, a thing long is produced, which is called a *Line*; then from the Motion of the Line ſprings a Superficie; and from the Motion of the Superficie, *ſomewhat thick or groſs*, which is alſo called a *Solid and Mathematical Body* There is no farther Progreſs; for which way ſoever a ſolid Body is moved, a ſolid Magnitude will be deſcribed. They therefore, who among the Kinds of Magnitudes rank Surdoſolids, Quadratoquadrats, *&c.* do idly; for theſe are not Figures, but only Numbers, that is, not Magnitude, but Magnitudes.

Motion, Time, Force, Weight, belong likewiſe to the Mathematicks; for one of theſe may be ſaid to be bigger or leſs than another: Nay in ſo far the Mathematicks treat of Qualities, as one may be called more or leſs ſuch than another. For of whatſoever may be adverbially ſaid, *more or leſs*, that hath a Foundation in ſome Magnitude. The Compariſon of Motions, Magnitudes, Times, *&c.* amongſt themſelves, pertains alſo to the Mathematicks; for by reaſon of the various *Inequalities* of Things, one *Inequality* is more or leſs than another. *Inequality* therefore hath its own Magnitude, and that is it which is uſually called *Proportion* or Reaſon. Number belongs to the Mathematicks alſo; for to what Science a Body, or Motion, or Time, or a Line belongs in the Singular; all theſe belong to the ſame in the Plural. For when One is nothing elſe but a *thing number'd*, Number can be nothing, other than *Things number'd.*

Now to ask how big a thing infinite is, would be an abſurd Queſtion, and not at all to be anſwer'd. For neither can a finite Magnitude be divided into Parts infinite in Number, nor can an infinite Number be given: when Mathematicians frequently ſay, *Let a Line be drawn out in infinitum*, they are not ſo to be underſtood, as if they thought that would be done, but that they leave it to be produced at the Reader's pleaſure.

The thing that is enquired of thoſe Subjects of the Mathematicks, is in a manner nothing elſe, than how great the propoſed Magnitude is, as it is compared with another Magnitude of the ſame Kind; or that I may ſay it in a Word, the Proportion is enquired. It is therefore neceſſary to one that ſtudies Mathematicks, exactly to underſtand theſe, and all other Terms which Mathematicians uſe; that is to ſay, to know the true and clear Definitions of the ſame.

Quantity (which the Antients no where defined, and who, for want of a Definition thereof, raſhly diſputing about the Nature of an Infinite, have erred) I thus define: Quantity is a Magnitude every way determined either by Expoſition, ſo that to one asking, How great is it? It may be anſwered, As great as you ſee; or by Compariſon, ſo that it may be anſwered, It is one or more Feet great, or any other ſuch way. But above all things, he ought to have an accurate Definition of Proportion, or *Mathematical Reaſon*; and beſides, to know how to Add, Subtract, Multiply, and Divide Proportions, and the Varieties of Compariſons; of which, though moſt of them be taught by *Euclid*, and his Interpreters, yet I ſhall reduce almoſt all of them to the next Chapter.

The Matter being known, and what we enquire into concerning it, the next thing is to take the Principles: The Principles of what? the Principles of the Science, or of knowing that we ſeek for. Now nothing but Truths can be known. The Principles then, of the Mathematicks, are the *firſt Truths*, which we are not taught, but know them by the Light of Nature, ſo ſoon as we hear them ſpoken. They are therefore called the firſt Propoſitions, becauſe of them the firſt of all Syllogiſms conſiſt. The Subject then of a Science, or any part of it, is not to be called a Principle; as a Point of Geometry, or an Unity of Arithmetick (which as a certain Maſter taught publickly) tho' the firſt Book of *Euclid*'s *Elements* begins with a Point, and the ſeventh Book with an Unity.

The Principles of Mathematicks, or firſt Propoſitions, are Definitions; but that theſe may be uſeful, they muſt not only be true, but likewiſe accurate, leſt that through ſo great Variety of Significations, we either proceed with Uncertainty, or by continual Diſtinctions being beat out of the Path, loſe the Truth by jangling. For as if a Hare ſhould ſtart amidſt a Company of Maſtiffs, the Dogs let looſe would fall upon one another, whilſt the Hare is running for it; ſo likewiſe in a Theatre of thoſe who diſpute for Victory, Truth flies away undiſcerned. And for this Cauſe (I mean, to remove Diſtinctions) wiſe Men have introduc'd Definitions.

But to define well, that is, ſo to circumſcribe with Words the Thing in Hand, and that clearly, and with as much Bevity as can be, that no Ambiguity be left, is a very difficult Task, and not ſo much a Work of Art, as of the natural Intellect. However we are eaſed of moſt part of this Difficulty by the Induſtry of the antient Geometricians; ſave that *Euclid*'s Definition of a Line needs ſome Correction, or at leaſt a ſound Interpretation. It is true indeed, that in comparing of Longitudes, there is no reſpect at all to be had to Latitude; but yet in the Conſtruction of Figures, Latitude is neceſſary; for without Latitude, it is impoſſible to deſcribe a Figure; nor can it be known in what part of a broad

Line that inviſible Line is to be underſtood. The reſt, tho' all of them be not moſt exact, yet becauſe they are true Propoſitions, they ſpoil not his Demonſtrations.

Now Definitions are of two Kinds, of which the one barely explains the Nature of the Thing, and the other the Cauſe or Manner of producing it. But theſe Definitions are moſt uſeful to the advancing of Knowledge, which contain the Cauſes of, and Manner of producing the Thing defined. For that Saying of *Ariſtotle*'s is true, *To know is to know by the Cauſe*: The reſt, which only declare the Eſſence of the Thing defined, are generally leſs fruitful; for nothing follows from them, that was not before contain'd in them; nor matters it, whether their Properties be called Definitions, or the Definitions, Properties.

Petitions are uſually reckoned among Principles alſo; but theſe are not the Principles of knowing, but of conſtructing, that is, of the Deſcription of Figures, wherein nothing is required, but that they be poſſible.

All Propoſitions likewiſe, whoſe Truth is obvious to the Light of Nature, are to be held for Principles. Now there no Doubt to be made of the Truth of a lawful Definition, becauſe they have their Truth from the Conſent and Will of Men, who, at their Pleaſure, give Names to Things explained. But there are ſome Propoſitions, which tho' they depend on Definitions, and may by them be demonſtrated; yet are ſo perſpicuous, that even without a Demonſtration they can force an Aſſent. Theſe are called *Axioms*; but care is to be had, that they be not held for true upon the bare Authority of Maſters. There is great Difference between the Authority even of the beſt Maſter, and the Light of Nature: For there is nothing ſo abſurd, but that ſome Maſter (in Defence of ſome Error) hath ſome time written Things as abſurd.

The Manner likewiſe of adding, ſubtracting, multiplying, and dividing of Coſſick Numbers (that is, Things number'd) is to be foreknown; which is eaſy, and taught by *Vieta* in his *Iſagoge*, but moſt clearly and briefly by *Oughtred*, in the *Key of Arithmetick*; from whence I have transferred as much as I thought fit into the third Chapter. The Nature and Manner of finding out Square and Cubick Numbers, with their Roots, is alſo to be learnt. This is handled in the Fourth Chapter.

The Nature of an Angle is likewiſe to be known; and therefore I have explained the Properties thereof in the Fifth Chapter.

To conclude, if he uſe any thing that is well demonſtrated by any Author for a Principle, he will do well; for every thing known ſtands inſtead of a Principle, for a farther Progreſs in Knowledge.

Some perhaps may ſay, that I place all Geometry in *præcognita*, or *Things to be foreknown*; but they are miſtaken: For he is a Geometrician (as I ſaid before) who knows how to determine a Quantity by Compariſon, amongſt Quantities of the ſame Kind. But thoſe Things which I would have foreknown, are only Properties that attend Geometry, and Steps whereby we are initiated in the Myſteries of ſublimer Geometry, and whereof we ſtand in need not only for Figures and Mechanick Works, but alſo for the Knowledge of all natural Cauſes, as being all contained in Motion.

The Principles and Theorems, which are well demonſtrated in the Books of the ancient Geometricians, being well underſtood (for I write to thoſe who deſire to improve the Science, the Thanks for the Work being due to the firſt Inventors; and no Man can reject what is already done, but who is very learned in his own Conceit) the next Thing is, (not to demonſtrate, but) to ſeek out the Cauſes of Proportion, that one Magnitude hath to another: For no Man can demonſtrate what he knoweth not, nor know but what he hath found out, nor (unleſs very ſeldom) find out what he hath not ſought after. There is no certain Art of Invention by Sagacity, trying, ſuppoſing, deducing Conſequences from Things ſuppoſed: Men often attain either to the Cauſes of the Matter ſought after, or to ſomething impoſſible. And by this Sagacity (not by Art) all the ancient Geometricians performed what they did, as manifeſtly appears, in that moſt of their Theorems begin with the Note of Suppoſition [*If*]. But (one may ſay) how ſhall one chance to take that ſuppoſed, which he ſtands moſt in need of? There is no certain Method whereby the moſt convenient Suppoſitions may be choſen; and therefore we muſt try and experiment by meaſuring with a Compaſs, and other Inſtruments, how near we can come mechanically to the Truth of the Thing ſought. For Inſtance, in ſeeking the Length of the Circumference of a Circle (all Men knowing, by the Light of Nature, that any Arch is bigger than its Chord) it is an eaſy matter to come very near the true Length. The remaining Work is no more but to excogitate the poſſible Cauſes of ſo great a Propinquity, wherein if they ſucceed, they have the Principle of a Demonſtration. For indeed, in all Doubts, Nature it ſelf ſets us upon various Suppoſitions. But moſt Geometricians enquire nothing now adays, but if it be *Euclid's*; tho' it be well known that the moſt noble Theorems were invented, and the moſt beautiful Fabricks of the whole World built long before the Time of *Euclid*. Now all our modern Geometricians, almoſt, imagine to find not only the Proportions of Quantities, but alſo of Motions, that is, the Cauſes of all natural Effects, in the ſole Extraction of the Numeral Roots, and that a falſe one too; and the Reaſon is, becauſe they are ſo taught.

The laſt thing in Geometry is, the efficient Cauſes of the thing ſought being found, to demonſtrate from them as it were backwards: I ſay, from the efficient Cauſes, becauſe the Cauſes of Numerical Properties are, for moſt part, nothing but arbitrary Impoſitions of Numeral Names; and Arithmetical Operations are the Demonſtrations of the Works themſelves.

CHAP. II.

Of Reaſon or Proportion.

1. REaſon is the *Relation* of a Magnitude to a Magnitude of the ſame kind, according to Quantity; as two Magnitudes, 2 and 5 being propoſed; the Reaſon of the firſt to the ſecond is the Relation of the Quantity of 2 to 5, that is, the Quantity of the Magnitude 2, as it is compared with the Magnitude 5: For the Inequality of two Magnitudes may be more or leſs than the Inequality of two other things unequal.

2. Therefore, for ſhewing Proportion or Reaſon, it is neceſſary that two abſolute Magnitudes be expoſed,

poſed, of which the former is uſually called the Antecedent, and the ſecond the Conſequent.

So that neither both Quantities together, as 2 and 5, nor one alone, as 2 or 5; neither $\frac{2}{5}$ (therefore our Algebraiſts do err, who ſay that Fraction and Reaſon are the ſame thing) neither the Difference betwixt 2 and 5, but the Quantity of Inequality that is betwixt two Magnitudes, is that which is called Reaſon or Proportion: For though both 3 and 2, and 9 and 8, differ only by a Unity, yet their Inequality is not the ſame. Yet if there be three Magnitudes, the leaſt, the mean, and the greateſt; the Reaſon or Proportion of the leaſt to the mean, is greater than the Reaſon of the ſame to the greateſt. But on the contrary, the Reaſon or Proportion of the greateſt to the mean, is leſs than the Reaſon or Proportion of the ſame to the leaſt. We might ſay that *Teucer* was bigger, compared to *Ulyſſes*, than to *Ajax*; and *Ajax* was leſs, compared with *Ulyſſes*, than with *Teucer*; the Reaſon of this is manifeſt; for by how much a leſſer Magnitude comes nearer a greater Magnitude, by ſo much it hath a greater Reaſon or Proportion to it; and by how much more a greater Magnitude exceeds a leſs, by ſo much it hath alſo to it a greater Reaſon or Proportion.

3. Now the Proportions or Reaſons of leſs to greater, and of greater to leſs, are different Kinds of Quantity; whereof the one is the Proportion of Defect, the other of Exceſs; of which the former, by how much the more it is multiplied, by ſo much the Defect becomes always the leſs; therefore it can never exceed or equal the Proportion of Exceſs.

4. Reaſons or Proportions are the ſame, or alike, or equal, in Numbers, when they are of the ſame

Denomination, or may be reduced to the ſame Denomination; as if both the firſt of the ſecond, and third of the fourth be $\frac{2}{1}$ or $\frac{4}{2}$: But where the Proportions are not expreſſible by Numbers, both the firſt of the ſecond, and third of the fourth, are correſpondent Sides of like Triangles, and are called Proportional, as having Portion for Portion, by the *Greeks* Ἀνάλογον, as the ſame thing ſaid again.

5. Continual Proportionals are ſuch, as, being proportional, have a middle Quantity common, as 1, 2, 4.

6. The Root of a Square Number is that Number, which, being multiplied by it ſelf, produceth ſome Number: Now the Number produced is called a Square or Quadrat, and the Root is always an aliquot Part of its own Square; as, if the Root be 8, the Square will be 64, and the Root the eighth Part of it.

7. It is then manifeſt, that the Root of a Square Number is not the Side of a Square Figure, though ſome Algebraiſts deny this, being now aſhamed to confeſs the Truth.

8. If there be three Magnitudes continually proportional, and from the ſame Antecedent three other continual Proportionals; the Proportional of the laſt to the laſt, will be the double of the Proportion of the ſecond to the ſecond; as is manifeſt in theſe Numbers, 2, 4, 8, and 2, 6, 18; the Proportion of 18 to 8, is the double of the Proportion of 6 to 4. For as 18 is to 8, ſo is 9 to 4; but 9 to 4 hath a double Proportion of 6 to 4. *Elem.* 14. *Prop.* 28.

9. Two Proportions are compounded or added together, when both Antecedents and Conſequents being multiplied among themſelves, a Proportion is made. For Example, if the Proportion of 2 to 3 be to be added to the Proportion of 4 to 5; let the An-

tecedents 2 and 4 be multiplied one by another, and they make 8; then let the Conſequents 3 and 5 be multiplied one by another, which make 15. This being done, the Proportion compounded of the Proportions of 2 to 3, and 4 to 5, is the Proportion of 8 to 15, according to *Eucl. Elem.* 6. *Def.* 5.

10. Otherwiſe, if it be thus; as 4 is to 5, ſo 3 to another which ſhall be $3\frac{3}{4}$, the Proportion of 2 to $3\frac{3}{4}$ (that is, both being multiplied by 4) will be of 8 to 15, that which is compounded of the Proportions of 2 to 3, and 4 to 5.

11. The Subtraction of a Proportion from a Proportion, is done by dividing the Quantities of the whole Proportion to be ſubtracted by the Antecedent, and the Conſequent of the whole to be ſubtracted by both. For Example, Let the whole Proportion be of 8 to 15, the ſubtracted of 4 to 5; I divide 8 and 15 by 4, the Quotients are 2 and $3\frac{3}{4}$. Again, I divide the Conſequent of the whole Proportion 15 to be ſubtracted by both the Quantities, to wit, 4 and 5, the Quotients will be $3\frac{3}{4}$ and 3, which are in the Proportion of 12 to 15, placing them in order, 8, 12, 15. If from the Proportion of 8 to 15, the Proportion of 12 to 15, that is, of 4 to 5, be taken; it is manifeſt that there remains the Proportion of 8 to 12, that is, of 2 to 3.

12. Otherwiſe, if it be thus; As the Conſequent of the Subtrahend, which is 5, is to its Antecedent 4; ſo the Conſequent of the whole, which is 15, is to the other 12; placing in order 8, 12, 15, it is plain that the Proportion of 12 to 15, that is, of 4 to 5, being ſubtracted from the whole Proportion of 8 to 15, there remains the Proportion of 8 to 12, that is, of 2 to 3.

13. Proportion is multiplied by ſaying, As the Antecedent is to the Conſequent, ſo the Conſequent to a third, and ſo the third to a fourth, *&c.*

14. A Proportion is divided by Number, when between the Antecedent and Conſequent, one or more mean Proptionals are interpoſed.

15. If there be never ſo many Magnitudes of the ſame Kind, the Proportion of the firſt to the laſt, is compounded of the Proportions of the firſt to the ſecond, of the ſecond to the third, and ſo forth unto the laſt; as in any Numbers, 4, 9, 10, 3, the Proportion of 4 to 3, is compounded of the Proportions of 4 to 9, and 9 to 10, and 10 to 3, which is demonſtrated by many.

16. But there is another Kind of compared Quantity, which is alſo called Proportion, not Simply, but Arithmetical Proportion. Now four Quantities are ſaid to be in Arithmetical Proportion, when the one conſtantly exceeds the other by equal Differences. Concerning which Proportion, I ſhall only ſay, *If a Line were cut into as many equal Parts, as it is intelligible it may be divided into, the Geometrical Proportional Mean between every next two of theſe Parts, will be the ſame with the Arithmetical Mean.* If (for Example) a given Line ſhould be divided into the Parts 2 and 16, the Arithmetical Proportional Mean will be 7; ſo then 2, 9, 16 are continual Arithmetical Proportionals. Now the Geometrical Proportional Mean is leſs than 6; but if the ſame Line had been divided into 100000 equal Parts, and both a Geometrical and Arithmetical Mean taken betwixt the Whole and its Parts 99999, how ſmall a Difference would be between thoſe two Geometrical and Arithmetical Proportions? Therefore by how much leſs the Difference

betwixt the Extremes is, by ſo much leſs is always the Difference between the Geometrical and Arithmetical Mean. Wherefore if ſuch Means were every where interpoſed, all their Difference would vaniſh. This is a moſt uſeful Propoſition for finding out the Proportions that ſtrait Lines have to crooked.

CHAP. III.

Of Algebraical Operations.

1. THE Algebraiſts inſtead of Things, that is, inſtead of Magnitudes, put Letters as the Species of Things, which in this manner we are taught to Add, Subtract, Multiply and Divide. If the Species be ſolitary, as A and B, they are added by interpoſing the Sign +, which ſignifies more. A and B then being added, the Sum is A + B; but if B be to be ſubtracted from A, they interpoſe the Sign —, which ſignifies leſs, or a Mulct. Therefore A — B is that which remains of the Magnitude A, after that B is taken from it: Multiplying by a Number, is when the Number Multiplier is prefix'd, as 2 A, 3 A, *&c.* The Multiplication of A by itſelf is performed by writing A A or A*q*; a Line cannot be multiplied by itſelf; and this is the firſt thing wherein the Algebraiſts of this Age differ from *Diophantus* and the Ancients, who contained themſelves within the Bounds of Arithmetick. If A be to be multiplied into B, they write AB, if it be written $\frac{A}{B}$, A is underſtood to be

divided by B, which ſignifies the Quotient, as if A be 10, and B 2; that Mark $\frac{A}{B}$ is 5; This does indeed, very well in Numbers; but if A A be a ſquare Figure, and B the Diviſor a Line; then that Quotient $\frac{AA}{B}$ ſhews how often the Line B is in the ſquare Figure A A, which is againſt the Nature of true Algebra, and likewiſe abſurd; for Planes or Flats are not compoſed of aggregated Lines.

But if the Species be affected with like Marks or Signs, as $+A+A$, they being added together make 2 A. For this is a true Rule of *Oughtred*, and the Algebraiſts; in Addition the Species are to be added, and a common Mark to be affixed. If therefore $-A$ be added to $-A$, the Sum will be $-2A$; but if the Marks be different, then this is the Rule; let Species be ſubtracted from Species, and the contrary Mark prefixed to what remains; ſo that $+A$ and $-A$ added together make 0, that is nothing.

3. But if two affected Species be to be added to two affected Species, this is the Rule: If they be like Signs, let the Species be added, reſerving the Signs. So $A+B$, and $A+B$ added together make $2A+2B$. But if the Marks be different, as in theſe $+A-B$, and $-A+B$, the Rule is; let the like affected be reduced under their proper Signs, and the Sum will be $+A+B-A-B$, that is nothing.

4. In Subtraction, if the Species be ſolitary, and each have the Sign +, as $+A+B$, and $+B$ be to be ſubtracted from $+A$, the Remains is the Species from which the Subtraction is made, with the Sign +, together with the Species ſubtracted having the con-

trary Sign. For B taken from A leaves A — B; but if both the Signs be —, the Difference of the Species with the Sign + is to be taken for the remaining Part, as if —5 be to be ſubtracted from —7, the Remains will be + 2. For it is manifeſt, that — 5 is more than — 7 by two Unities. And that it is ſo, is eaſy to be underſtood by any that will take the Pains to conſider it. For what is it elſe, to take the Defect from a given Magnitude, but to add to it the deficient Magnitude which it wanted? If therefore — A be to be taken from A, it is manifeſt that there remains 2 A; for by the very taking away of the Defect, the Acceſſion of that which was wanting is added, in the ſame manner as he that remits a Debt, beſtows upon his Debtor as much as he hath remitted. If ſeveral Species affected with Signs be to be multiplied by one or more Species affected, this is the Rule: If the Signs be alike, let the Sign + be prefixed to the multiplied Species; but if different, the Sign —; as in this Example,

$$\begin{array}{rcr} A+B & \qquad & A-B \\ A+C & & A-C \\ \hline +CA+BC & & -CA+BC \\ AA+AB & & +AA-AB \\ \hline AA+CA+AB+BC & & +AA-CA-AB+BC \\ \hline \end{array}$$

Which is true indeed in Numbers; but if they be Lines, falſe. For nothing can be multiplied but by a Number.

As many Species as you pleaſe, and howſoever affected, are underſtood to be divided, when they place the Diviſor under the Dividend, as $\frac{A+B}{B-C}$ is the Quotient of A+B divided by B—C, that likewiſe is true in Numbers; but if A+B be a Superficie, and B—C a Line, it is abſurd, as we obſerved before. What remains is no more but Labour in ſeeking out a commodious Suppoſition, from whence they may begin, which is not Art, but groping and chance. Now when they have found out any Proportion by changing, converting, multiplying, dividing, compounding, and elevating, and depreſſing it by a Scale of continual Proportionals, they toſs the ſame, until that at length unawares, and by hap-hazard, they ſtumble upon ſome Equation that may ſatisfy an Arithmetical Problem: But it is impoſſible by this Method to anſwer a Geometrical Queſtion. Algebra is, indeed, one of the Rules of Arithmetick, to the attaining the Theory whereof, two or three Days at moſt are required, though to the Promptitude of Working, perhaps the Practice of three Months is neceſſary.

CHAP. IV.

Of Square Figures, and Square Numbers.

1. A Square Figure, and a ſquare Number, when they have ſo many ſquare and equal Parts, are one and the ſame thing; for a whole Figure, and the Number of its Parts are the ſame.

2. If ſquare Figures be deſcribed from *creſcent Radii* (or Radius's increaſing) as Numbers in natural Order from an Unity, they differ among themſelves as odd Numbers of the ſame Order. For the leaſt Figure is one, to this the next odd Number (which is 3.) being added, makes the next ſquare Figure (4.) To this again, the next odd Number (5) being added, makes the third Square (9) and ſo on. Therefore the Exceſſes of the ſaid Squares increaſe as odd Numbers from an Unity, to wit, 1, 3, 5, 7, *&c.*

3. If there be three ſquare Numbers next to one another in order, and a Unity be taken from the middle, the Number that is left is the mean Proportional betwixt the Extremes. For Example, 100, 81, 64 are ſquare Numbers next to one another in order. Let the *Unity* be taken from 81, there remains 80, which is a mean Proportional betwixt 100 and 64. In like manner 49, 36, 25, are ſquare Numbers next to one another in order. Take an Unity from the middle 36, there remains 35 the mean Proportional betwixt 49 and 25. So in theſe 9, 4, 1; if from the middle 4 one be taken, there remains 3 the mean betwixt the Squares 9 and 1. Laſtly, 400, 361, 324, are Squares next to one another in order: If from the middle 361 an Unity be taken, there remains 360, the Mean betwixt the Extremes 400 and 324, and ſo in all.

4. Between two next ſquare Figures, if to the leaſt Square its Root be added, the whole will be the mean Proportional. For if to the Square 1, its Root (which is 1) be added, the whole 2 is the mean Proportional betwixt 1 and 4. In like manner, if to the Square 4 its Root be added, the whole 6 is the mean Propor-

tional betwixt 4 and 9; and ſo in all, as is manifeſt by Induction.

5. If from a Square its Root be taken, there will remain the Mean betwixt it and the next leſſer Square. Let 100 be a ſquare Figure, if its Root 10 be taken from it, there will remain ninety, the Proportional Mean betwixt 100 and 81, the next leſſer Square. In like manner if from the Square 9, its Root 3 be taken, the remaining 6 will be the Mean betwixt 9 and the next inferior Square 4, and ſo always.

6. But if the Squares be not next to one another, but that one interpoſes, the leſs Square with two Roots, is the mean Proportional betwixt thoſe two Squares. For if to the Square 1 its two Roots, that is 2 be added, there wlll be 3, the Proportional Mean betwixt 1 and 9. Likewiſe if to the Square 4 its two Roots, that is 4, be added, there will be 8, the Mean betwixt 4 and 16, and ſo perpetually. The ſame Mean will be had, if from the greater Square, its two Roots be ſubtracted. For if from 16, its two Roots be taken, there will remain 8, the Mean betwixt 16 and 4.

If laſtly, Two Squares have other two between them, and to the leis Square, the Triple of its Root be added, you will have the Mean betwixt the two extreme Squares. For if 3 be added to the Square 1, there will be 4, the Mean betwixt 1 and 16; or if to the Square 4, the Triple of its Root, to wit, 6, be added, there will be 10, the Proportional Mean betwixt 4 and 25; or if from 25 the Triple of its Root, to wit, 15, be taken, there will remain 10, the Mean betwixt 25

4, and ſo always. From which follows a general Rule, that the mean Proportional betwixt two Squares, is ſo many Roots of the leſſer added to the leſſer; or what remains after ſo many Roots are taken from the greater, as the greater Square is diſtant Places in the natural Order of Numbers from the leſſer.

7. Hence it is manifeſt, that the Root of a Square Number is not (as Algebraiſts imagine) the Side of a ſquare Figure, nor at all a Line, becauſe a Line adds nothing to a Flat, or Plane.

8. If two Squares are next to one another, both taken together, are greater than the double Mean, by a Unity, or one ſquare Particle. For 1 and 4 are next to one another, both together make 5, the double of the Mean is 4, the Difference is 1. In like manner 100 and 121 are next, both together are 221, the Mean is 110, the double of the Mean 220, the Difference 1, and ſo in all.

9. If Squares admit of one between them, the Extremes taken together, are greater than the double Mean by four; for 1 and 9 admit of one betwixt them, both together make 10, the double Mean is 6, the Difference 4. Likewiſe 100 and 144 receive one between them; the Extremes put together are 244, the Mean is 120, the double Mean 240, the Difference 4, and ſo every where.

10. If between two Squares, other two interpoſe, the Extremes together are greater than the double Mean by the Number Nine. For 1 and 16 have two betwixt them, added together they make

17, the double Mean is 8, the Difference 9. Likewiſe 100 and 49 have two betwixt them, put together they make 149, the Mean is 70, the double of that 140, the Difference 9. In like manner, three being interpoſed, the Extremes exceed the double Mean by ſixteen, and ſo forth by ſquare Numbers throughout the whole Series of Squares.

11. Betwixt two Cubick Numbers, two mean Proportional Numbers intervene: Now they are found out after this manner; if the Cubes be next to one another, let the leſs Cube be divided by its Root, then let the Quotient be multiplied by a Number that exceeds the Root by an Unity, the Product will be the leſs of the Means. For Example, Take the next Cubes 1 and 8; let the leſſer Cube 1 be divided by its Root 1, the Quotient will be 1, then multiply 1, by one more 1, that is 2. This Number 2 is the leſs of the Means betwixt 1 and 8; by the ſame Method the greater Mean is found out, 2 being divided by 1, the Quotient is 2; then 2 being multiplied by one more 1, makes the greater Mean 4. Laſtly, 4 being divided by 1, the Quotient is 4, and 4 being multiplied by one more 1, it makes 8; or let the leſs Cube be 64, its Root 4, the next greater Cube 125; 64 being divided by 4, the Quotient is 16; 16 being multiplied by 4 more 1, makes 80, the firſt of the Means. Again, 80 being divided by 4, the Quotient is 20; which multiplied by 4 more 1, makes 100 the greater Mean. In the laſt Place, if 100 be divided by 4, the Quotient will be 25, which being multiplied by 4 more 1, makes 125.

12. If two Cubick Numbers have one Cubick Number betwixt them, let the leſs Cube be divided by its Root, and let the Quotient be multiplied by the Root more 2, the Product will be the leſſer Mean; let the Cubes be 1, 27: 1 being divided by 1, the Quotient is 1, which multiplied by 1 more 2, makes 3 the leſſer Mean. Again, 3 divided by 1 is 3, then 3 multipled by 1 more 2, makes 9 the greater Mean. Laſtly, 9 divided by 1 is 9, which being multiplied by 1 more 2, is 27.

13. In like manner, if the Cubes be 8 and 64, 8 being divided by its Root 2, the Quotient is 4, and 4 multiplied by 2 more 2, makes 16, the firſt Mean; and 16 being divided by 2, makes 8, which multiplied by 2 more 2, makes 32 the ſecond Mean. To conclude, 32 being divided by 2, the Quotient is 16, which multiplied by 2 more 2 makes 64. In like manner, if the Cubes be 27 and 125; 27 being divided by 3, the Quotient is 9, which multiplied by 3 more 2 makes 45 the firſt Mean. Again, 45 being divided by 3, the Quotient is 15, which multiplied by 3 more 2 is 75, and 75 divided by 3, the Quotient is 25; and this multiplied by 3 more 2 makes 125, and ſo always in Cubes that intermit one.

If two Cubick Numbers intermit two Cubick Numbers, the leſs being divided by its Root, the Quotient multiplied by the Root more 3, will give the firſt Mean. Let 27 and 216 be two Cubes intermitting two Cubes (to wit 64 and 125) 27 being divided by its Root, the Quotient is 9, which being multiplied by the Root 3 more 3, makes 54 the firſt Mean; 54 divided by 3 makes 18, which being multiplied by 3 more 3 is 108. Laſtly, 108 being divided by 3, the Quotient will be 36, which multiplied by 3 more 3, makes 216.

If two Cubick Numbers intermit three Cubick Numbers, and the leſs Cube be divided by its Root, and the Quotient multiplied by that Root more 4, the Product will be the firſt of the Means. And ſo through all the Cubes of Numbers that are in a natural Order, adding 5, 6, 7, *&c.* you may find the firſt Means betwixt any two Cubes whatſoever.

* * *

CHAP.

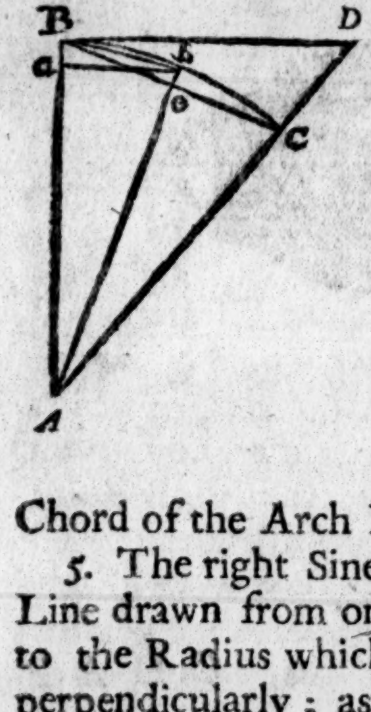
CHAP. V.

Of ANGLES.

1. A Strait or right Line Angle, is the Digreſſion of two ſtrait (or right Lines) from the ſame Point in the Circumference.

2. The Quantity of a ſtrait Line Angle, is the Quantity of the Proportion that the intercepted Circumference hath to the Perimeter of the Circle.

3. The Quantity then of an Angle is not a Line, neither a Superficie, nor yet a ſolid Body, but the Quantity of the Proportion which the Revolution of the *Radius* hath to another Revolution of the ſame.



4. The Chord of an Angle, Circumference or Arch, is a ſtrait Line that joins together any two Points of the Circumference; as if there be ſtrait Lines from the ſame Point AB, A*b*, AC, and the Angle ABD a right Angle; but the Arch B*b*C. Then the ſtrait Line BC is the Chord or ſubtenſe of the Arch BC, and the ſtrait Line B*b*, the Chord of the Arch B*b*.

5. The right Sine of the Circumference is a ſtrait Line drawn from one Term of the Circumference, to the Radius which paſſeth thro' the other Term perpendicularly; as B*e* or *ab*, is the right Sine of the Arch B*b*.

6. The Secant of an Arch or Circumference, is a ſtrait Line drawn from the Centre by one Term of the Arch to the ſtrait Line which is drawn perpendicularly in the other Term to the Radius; as A D is the Secant of the Arch B C.

7. The Tangent of an Arch, is that Part which is intercepted of the right Line which is drawn perpendicularly from the Beginning of the Arch; as B D is the Tangent of the Arch BC, or of the Angle ABC.

8. Hence it is manifeſt, that the right Sine of any Arch, is the half of the Chord of the double Arch, or that the Chord of the Arch, is equal to the double right Sine of the half of the Arch, as if the Arch B*b* be the half of the Arch B C, *ab* doubled will be equal to the Chord B C.

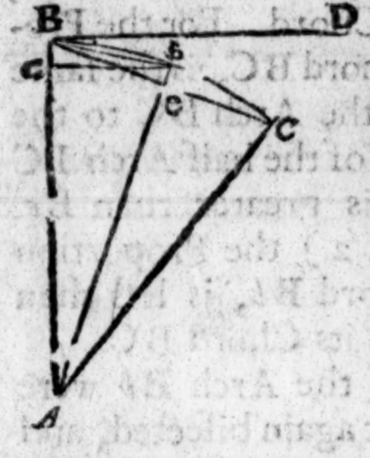
9. A leſſer Arch hath a leſs Proportion to its Chord, than a greater Arch hath to its Chord. For the Proportion of the Arch BC to its Chord B C, is the ſame with that of B*b*, the half of the Arch B C to the half Chord B*e*. But the Chord of the half Arch BC is the ſtrait Line B*b*, which is greater than B*e*. Therefore (by Chap. 1. Num. 2.) the Proportion of the leſſer Arch B*b* to its Chord B*b*, is leſs than that of the greater Arch B C to its Chord B C.

10. Hence it follows, that if the Arch B*b* were biſected, and the Biſegment of it again biſected, and ſo on as often as can be done; we would at length come to a Segment of an Arch, the Exceſs of which upon its own Chord ſhould be leſs than any given Longitude, and by conſequence, leſs than the Breadth of any Line having never ſo little Latitude.

11. In like manner, if a ſtrait Line were biſected, and the Biſegments thereof again biſected as often as might be, we would come at length to a Segment

leſs than any aſſignable Quantity, and therefore leſs than the Breadth of a Line having never ſo little Latitude.

12. Now that univerſal Saying, *Every Arch is greater than its Chord*, would hold univerſally true, if there could be given an Arch of a Circle, and a ſtrait Line, that had no Breadth; but neither is there a Line without Latitude, nor if there were, or could be but ſuppoſed, could it enter into a Demonſtration, that handles the Proportion of Superficies and Solids; becauſe they are different kinds of Quantity, as appears by the fifth Definition, El. 5. of *Eucl.* Neither (as I conceive) would Dr. *Wallis* have thought otherwiſe but that that Definition is not found among the Principles of Algebra in *Oughtred*'s *Key of Arithmetick*.



13. In any Triangle, if the Angle be cut in two equal Parts, and the ſtrait Line that cuts the Angle, cut alſo the Baſe; the Segments of the the Baſe will in Proportion of the Sides. This is clearly demonſtrated in *Eucl.* Elem. 6. Prop. 3.

14. There is another Kind of a plain Angle, which they call the Angle of Contact, ſuch as the Angle that the ſtrait Line BD makes with the Arch BC in the Point B. Which *Euclid* (or his Interpreter *Theon*) and many that have followed him, have affirmed to be greater indeed, than any acute Angle; and the Angle which AB makes with the ſame Arch BC, bigger than any acute Angle, yet leſs than a right, whieh is manifeſtly abſurd. For

then, as Dr. *Wallis* will have it, the Angle of Contact had been of the ſame Kind with the Angle that is made by the Revolution of the Radius; for every Angle that is not obtuſe, is either right, or leſs than a right Angle, by ſo much as any Angle is acute.

They were in the firſt Place miſtaken, in that they thought the Space betwixt the ſtrait Line BD, and the Arch BC to be an Angle, which (by Numb 3. of this Chapter) is falſe. They were alſo miſtaken, in that they took thoſe Angles for Quantites of the ſame Kind, which is falſe. For an Angle of Contact, howſoever multiplied, can never equal nor exceed an Angle made by the Circulation of the Radius.

15. If it were true, that the Angle made by the right Line AB, with the Arch BC, were leſs than a right Angle; it might come to paſs that an acute Angle increaſing uniformly, would at length become greater than a right, which yet could never equal it: The Ground of which Abſurdities is, that they thought no Line to have any Latitude, nor a Sine any Quantity at all.

16. That an Angle in the Centre is the double of an Angle in the Circumference (if they inſiſt upon equal Arches) is demonſtrated by *Eucl. El.* 3. *Prop.*20. But it is to be obſerv'd, that to the Eſſence of an Angle in the Circumference, it is not required that both the Sides ſhould terminate in the Circumference; for DBC is an Angle in the Circumference, though the Point D be not in the Circumference.

CHAP.

CHAP. VI.

Of the Proportion of the Perimeter to the Radius of the Circle.

1. A Quadrantal Arch is equal to the Radius, together with the Tangent of 30 Degrees.

Let the Radius be AB, the half Radius BE, or AF, draw the Line then EF, the Rectangle ABEF, will be the half of the Square from the Radius AB. By the Radius A B let the Arch BG be deſcribed, cutting EF in G: The Line then AG being drawn, and produced to B E produced in H, B H will be the Tangent of 30 Degrees, and BG the third Part of the Quadrantal Arch deſcribed by AB; then from the Point G to the Side A B, let the Perpendicular GI be drawn. In B A produced, take A N the double of AI, ſo that IN be the Triple of IA; then draw the Secant NG, cutting AF in T, and produce it to BH in *q*.

I ſay, that the ſtrait Line E*q* is equal to the Arch B G.

Upon A T deſcribe a Quadrantal Arch S T, becauſe then the Triangles N I G, N A T are alike; A T will be two Thirds of the ſtrait Line G I, the half Radius; that is, the third Part of the whole Radius.

Becauſe therefore AT is the third Part of the Radius, the Arch S T will be equal to the Arch B G. Let B G be cut in two in the Middle in *i*, and A T in M, and let the Perpendicular *i b* be drawn to AB, and A M will be the 6th Part of the Radius A B, and the Arch Y M equal to the Arch B *i*; and as N *b* is to N A, ſo ſhall *b i* be to A M: Draw then and produce N M, it will paſs thro' *i*; now let it be produced to B H in Q. Let *b i* be produced to the Concourſe with M *q* in *t*, and *b t* will be the Double of *b i*, and equal to the Chord of the Arch B G. For the right Sine of the Half of any Arch, is the Half of the Chord of the double Arch. Again, let the Arch B *i* be biſected in *e*, and A M in L; and draw the Perpendicular *e a* to A B, and *e a* will be the Half Chord of the Arch B *i*, and the 4th Part of the 2 Chords of the Arches B *i* and *i* G. Now let A L alſo be the 4th Part of the ſtrait Line A T; draw then and produce the Line N L, and it will paſs by *e*. Now let it be produced to B H in P. Therefore the Arch K L is equal to the Arch B *e*. In like manner, if M T be divided in X, draw the Line N X, and produce it, it will cut the Arch B G in *o*, ſo that *o* G is the 4th Part of the Arch B G, and the Quadrantal Arch V X will be equal to the Arch B *o*; and ſo perpetually. The ſtrait Lines then, which are drawn from the Point N, will divide both the ſtrait Lines A T, B *q*, and the Arch B G, into the ſame Proportions.

If then the Biſection were continued as much as may be, we would at length come to the Arch of a Circle, leſs than the Latitude of the Line or Side A B. Alſo according to *Chap.* 5. *Numb.* 9. the Proportion of an Arch to the Chord always decreaſeth. Wherefore the Difference of the Arch to its own Chord decreaſeth alſo, until it become leſs than any aſſignable Difference.

Let us then ſuppoſe the Latitude of the Radius AB to be equal to the leaſt Segment, or leſs than it (for by biſecting, it can never be reduced to nothing) the laſt and leaſt Segment will be in the ſtrait Line BH to *q*, and ſo continually all the equal Segments of the Arch BG will lie extended toward the ſtrait Line B*q*; for B*q* is greater than *bt*, which is equal to the Chord of the Arch BG. Let *ae* be produced till it meet with N*q* in *u*, and *au* will be equal to 4 Chords of the Arch B*e*, but leſs than the whole B*q*. In like manner, if the Arch *ae* be biſected, and the right Sine of it drawn, that produced till it meet the ſtrait Line N*q*, it will be equal to 8 Chords of the Arch *ae*, but leſs than the ſtrait Line B*q*; and ſo always, until one come to the laſt and leaſt Segment, whoſe Longitude is equal to the Latitude of the Radius AB.

Therefore, if the leaſt Latitude be allowed to the Side AB, the Arch BG, and ſtrait Line B*q* will be equal; but if all Latitude be denied to a Line, not only the leaſt Part of an Arch will be greater than its Chord; but alſo all its Parts to all the others, ſo that no Arch can be equal to a ſtrait Line. Neither can there be any Line drawn, nor Sine, nor Arch, nor Figure. For a Line without Latitude is nothing, and the Half of it as much.

Therefore B*q* is equal to the Arch BG, that is to the third Part of the Quadrantal Arch deſcribed by the Radius AB.

Now becauſe the Triangles GTF, EG*q*, are Rectangular and alike, and TF the third Part of the Half Radius AF; E*q* will be the third Part of EH (the Exceſs of the Tangent BH above the Half Radius BE). The Triple then of B*q* (that

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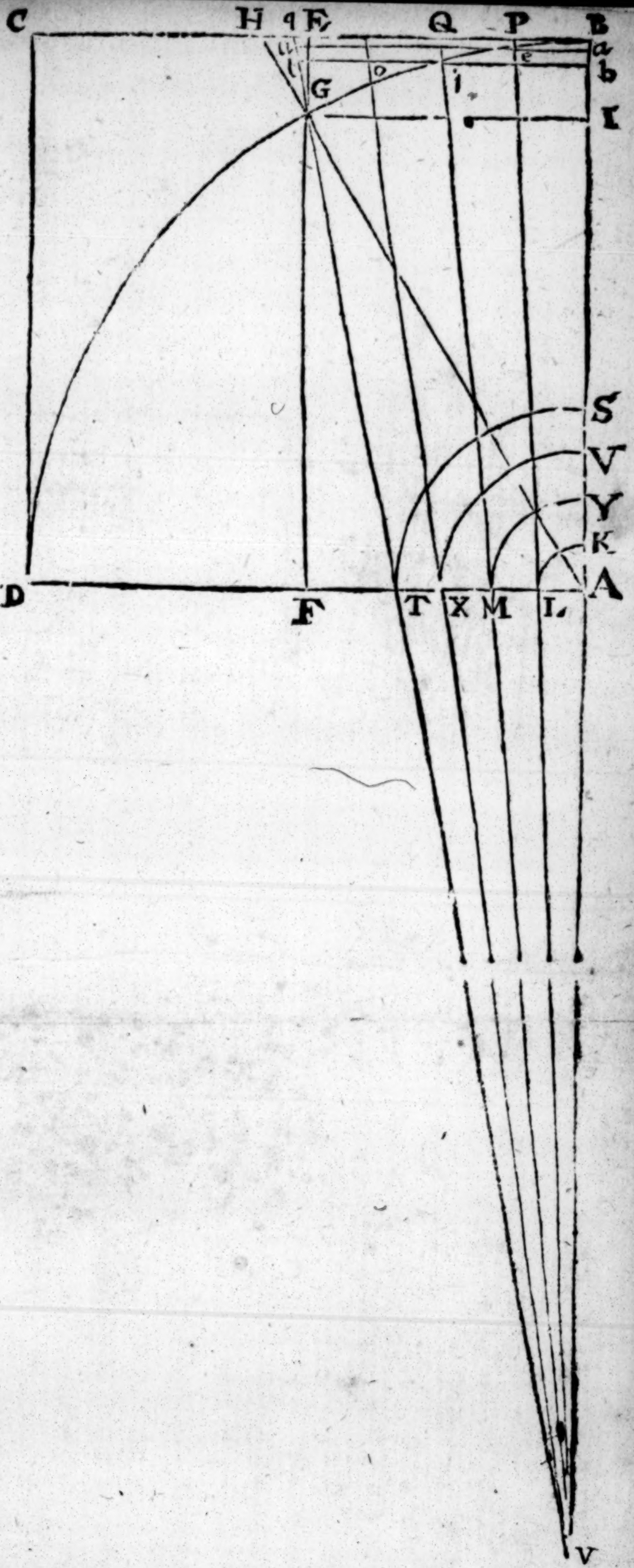

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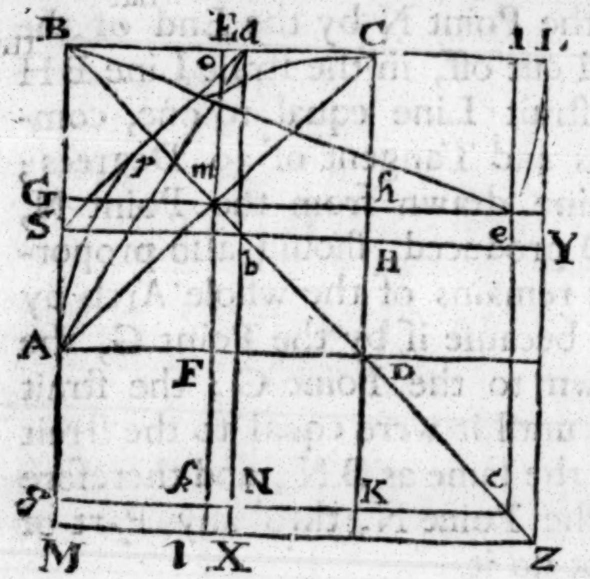
is, the Quadrantal Arch deſcribed by the Radius A B) is equal to the Triple of the Semiradius, together with the Triple of E *q*; that is, to the Radius A B, together with the Tangent of 30 Degrees, that is, together with B H, which was to be demonſtrated.

2. If B *q* were produced until it were tripled, and A F until it were doubled, it is manifeſt that a ſtrait Line drawn from the Point N by the End of the doubled A F, would cut off, in the ſtrait Line B H being produced, a ſtrait Line equal to one, compoſed of the Radius and Tangent of 30 Degrees; but that a ſtrait Line drawn from the Point N, paſſing through A D produced, ſhould alſo proportionally divide what remains of the whole Arch by B G, is impoſſible; becauſe if by the Point G, the Tangent were drawn to the Point G; the ſtrait Line G A produced until it were equal to the ſtrait Line B N, would do the ſame as B N; and therefore ſtrait Lines from the Point N, thro' any Part of the Arch, would not do it.

SCHOLIUM.

3. Becauſe I have ſhewed in *Book* 2. (of *Geometrical Roſes*) that a Quadrantal Arch is equal to a right Line, whoſe Square is ten times as great as a Square from the half Radius; and have now made out, that the ſame Arch is equal to a ſtrait Line, made up of the Radius and Tangent of 30 Degrees: In this Place I will, from moſt clear Principles, demonſtrate, that a Square from that ſtrait Line compoſed of the Radius, and Tangent of 30 Degrees, is alſo the Decuple of a Square from the Semiradius.

Deſcribe a Square ABCD, and let it be divided 4 Ways, as well by the right Lines EF, GH, as the Diagonals AC, BD, concurring in the Centre of the Square at *m*; and deſcribe the Arch AC cutting EF in *o*; and let it be produced to the Side BC in *a*. B*a* is then the Tangent of 30 Degrees; let the Side BG be produced, and in it being produced, let CI equal to the half Side BE, and CL equal to the Tangent B*a*, be taken. Then deſcribe a Square from the whole BL, which is BLZM, which is to be demonſtrated to be equal to ten Squares from BE or CI.



The Square from BE is BE*m*G, the Square from the Tangent B*a*, is B*ab*S, to which let CLYH*h* be equal.

Deſcribe alſo a Square from BI, which ſhall be BI*cg*, and let CD, EF be produced to the Side MZ cutting *gc* in K and *k*. Let *ab* alſo be produced to the Side MZ in X, and SH to LZ in Y.

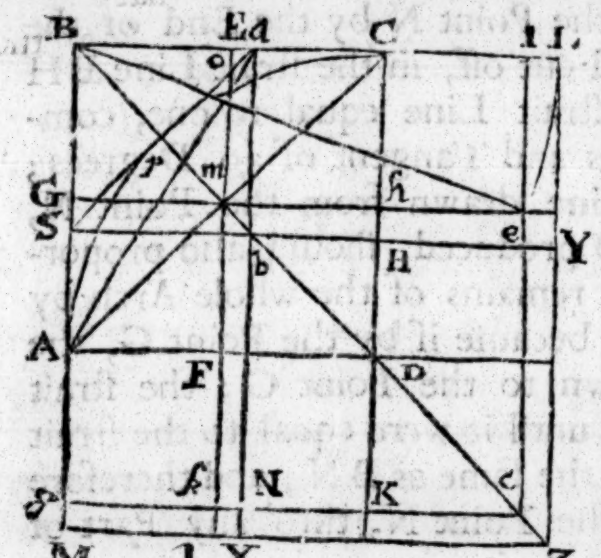
Underſtand now, that the 4 Sides of the Square ABCD, are each divided into 12 equal Parts. So it will be that the Square ABCD does contain 144 equal little Squares.

Becauſe the Side BC is 12, the Side BI is 18, the Square of the former is 144, of this 324, the Square from 19 is 361, the Square from 20 is 400; and theſe ſquare Numbers are in Order next to one another. Wherefore (according to *Chap.* 4. *Numb.* 3.) an Unity being taken from the mean Square 361, the remaining 360 is the mean Proportional betwixt 400 and 324, that is, the Root of the ſquare Number from 400 multiplied into 324.

Again, Becauſe BC is 12, the half Side BE is 6, which ſquared is 36. Now the Square from 7 is 49, the Square from 8 is 64; and theſe Squares are next to one another in Order. Wherefore (according to the ſame *Number* of the ſaid 4th *Chapter*) take an Unity from the mean Square 49, there will remain 48, the mean Proportional betwixt 64 and 36, or the Root of the Number 64 multiplied into 36. Now 48 is the Square of the Tangent of 30 Degrees in the Square 144, becauſe then 360 is the Decuple of the Square from the half Side BE, it is to the Square 144, as 10 to 4; ſo the Tangent of 30 Degrees, in the Square 360, will be to the Square of the Tangent alſo, as 10 to 4; but the Tangent of 30 Degrees, in the Square 360, to wit, the 3d Part of it, is 120, which Number is to 48, the Square of the Tangent in the Square 144, as 10 to 4. Therefore 360 is the Square from the right Line compoſed of the Radius and Tangent of 30 Degrees, which right compounded Line is BL, which was to be demonſtrated.

4. In the right Line I*c*, take I*e* equal to the ſtrait Line CI, which being joined to BE, the Arch of the Circle deſcribed by the Radius B*e* will paſs by L; for BI hath the Power of 9 Squares, whereof I*e* hath the Power of 1; therefore B*e* hath the

Deſcribe a Square ABCD, and let it be divided 4 Ways, as well by the right Lines EF, GH, as the Diagonals AC, BD, concurring in the Centre of the Square at *m*; and deſcribe the Arch AC cutting EF in *o*; and let it be produced to the Side BC in *a*. B*a* is then the Tangent of 30 Degrees; let the Side BG be produced, and in it being produced, let CI equal to the half Side BE, and CL equal to the Tangent B*a*, be taken. Then deſcribe a Square from the whole BL, which is BLZM, which is to be demonſtrated to be equal to ten Squares from BE or CI.



The Square from BE is BE*m*G, the Square from the Tangent B*a*, is B*ab*S, to which let CLYH*b* be equal.

Deſcribe alſo a Square from BI, which ſhall be BI*cg*, and let CD, EF be produced to the Side MZ cutting *gc* in K and *k*. Let *ab* alſo be produced to the Side MZ in X, and SH to LZ in Y.

Underſtand now, that the 4 Sides of the Square ABCD, are each divided into 12 equal Parts. So it will be that the Square ABCD does contain 144 equal little Squares.

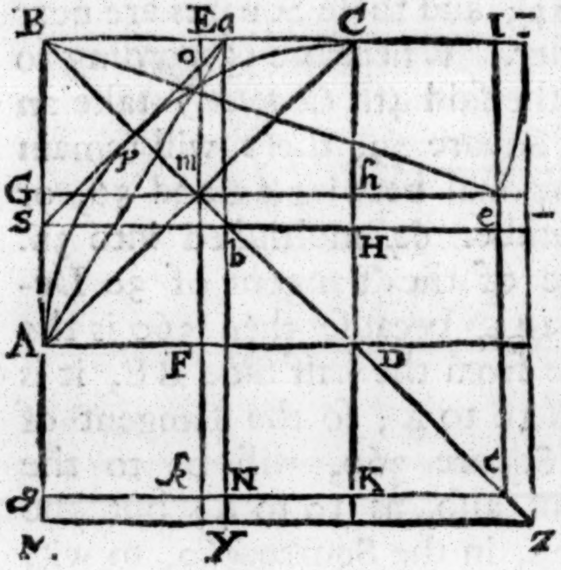
Becauſe the Side BC is 12, the Side BI is 18, the Square of the former is 144, of this 324, the Square from 19 is 361, the Square from 20 is 400; and theſe ſquare Numbers are in Order next to one another. Wherefore (according to *Chap.* 4. *Numb.* 3.) an Unity being taken from the mean Square 361, the remaining 360 is the mean Proportional betwixt 400 and 324, that is, the Root of the ſquare Number from 400 multiplied into 324.

Again, Becauſe BC is 12, the half Side BE is 6, which ſquared is 36. Now the Square from 7 is 49, the Square from 8 is 64; and theſe Squares are next to one another in Order. Wherefore (according to the ſame *Number* of the ſaid 4th *Chapter*) take an Unity from the mean Square 49, there will remain 48, the mean Proportional betwixt 64 and 36, or the Root of the Number 64 multiplied into 36. Now 48 is the Square of the Tangent of 30 Degrees in the Square 144, becauſe then 360 is the Decuple of the Square from the half Side BE, it is to the Square 144, as 10 to 4; ſo the Tangent of 30 Degrees, in the Square 360, will be to the Square of the Tangent alſo, as 10 to 4; but the Tangent of 30 Degrees, in the Square 360, to wit, the 3d Part of it, is 120, which Number is to 48, the Square of the Tangent in the Square 144, as 10 to 4. Therefore 360 is the Square from the right Line compoſed of the Radius and Tangent of 30 Degrees, which right compounded Line is BL, which was to be demonſtrated.

4. In the right Line I*c*, take I*e* equal to the ſtrait Line CI, which being joined to BE, the Arch of the Circle deſcribed by the Radius B*e* will paſs by L; for BI hath the Power of 9 Squares, whereof I*e* hath the Power of 1; therefore B*e* hath the

Power of 10 half Sides, the Arch therefore deſcribed by it will paſs by L.

5. We muſt not here paſs over a Difficulty, which in the 23d *Chapter* of the *Book* of *Principles*, I did indeed mark, but thought fit to leave the Solution of it to the Readers: The Solution of this will much conduce to the illuſtrating and facilitating of Geometry. This is the Difficulty: The Square CLYH (which is the Square of the Tangent of 30 Degrees, and equal by Conſtruction to B*a b*S) is to the Square CI*e b*, as 4 to 3, as is commonly known: Becauſe therefore the Gnomon ILZ*c*M ought to be equal to the Triple of the Gnomon ILY*e b*H, and ſo that Gnomon being tripled and placed upon this, ought to agree with it, but it agreeth not, for it is leſs. Hence it may ſeem concluſive, that the Side of the Square, which is the Decuple of the Square from the half Side, is greater than the Square from BL; but why? becauſe it is leſs by ſo much as the double little Square *m b*, or *c*Z, or *e*Y.



For ſuppoſe the right Line BC to be divided into 12 equal Parts, the whole Square ABCD will be 144 of thoſe Parts; the Square CIYH 48, the Square alſo AX 48, and the Square DZ 48, all theſe make 288. Now ſeeing the Square from the half Side C I is 36, the whole Square (which is 10 times as much) ought to be 360; 288 being then taken from 360, the Remainder 72 ought to be equal to the 2 equal Rectangles X D and D Y, and either of them ought to be 36, but it is not ſo. The Square F D K *k* is indeed equal to the Square from BE or CI (that is 36) from which if the Rectangle F *l* be taken, and the Rectangle X K added (ſeeing that is greater than this by the Quantity of the Square *k* X) the remaining Rectangle X D will be leſs than the Square F K, that is, than 36, as much as the Square *k* X or *c* Z; for the ſame Cauſe the Rectangle D Y is alſo leſs than 36 by the Quantity of the Square *e* Y or *c* Z. Therefore the whole Square B L Z M is leſs than the Decuple of the Square from the half Side, which is 36, by the Quantity of the double little Square *c* Z.

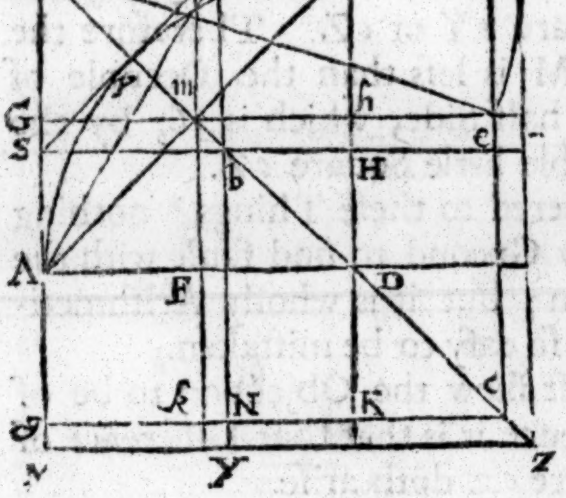
What can be anſwered to theſe Things? nothing truly, if there be any Ground to find fault with the former Demonſtration; but it is wholly Arithmetical, wherein it is not ſo eaſy to be miſtaken.

I will therefore firſt ſhew the Objection to be of no Force; then, whence it is that that Difference of the double little Square *c z* doth ariſe.

That it is of no Force, is manifeſt from hence, that by the ſame Argument it may be proved (it being ſuppoſed, that the right Line B C is divided into 12 equal Parts) that the Square of it alſo is leſs than 144, by the Quantity of the ſame double Square *c* Z. For ſeeing the Square C *m* is 36, the right Angle C *a b* H will be leſs than it by the Quantity of the Square *b m* or *c* Z. For from the Square C *m* on the one Side, the Complement E *b* is taken, on

the

the other *b b*; but E *b* is greater than *b b* by the Square *b m*. Therefore the Rectangle C *a b* H is leſs than 36, by the ſame little Square *c* Z; for the ſame Reaſon the Rectangle A *b* is leſs than 36, by the ſame little Square *c z*; there remains for compleating the whole Square, the Square *b* D, which is the Half of the Square from the Tangent B *a*. For join *a s* cutting the Diagonal in *p*, it will be as S *a*, or B *b* to B *a*, ſo B *a* to *b p*, that is, to H *b*; *a b* is then the mean Proportional betwixt B *b* and *b* H; therefore the Square *b* D is the Half of the Square B *a b* S, that is 24; therefore the whole Square 144 is equal to 48 + 24 + 36 + 36, leſs the double Square *c* Z, which is abſurd. For it is not to be doubted, but that the Square A B C D is 144. Therefore this Objection is of no Force, and has this Fault beſides, that the Square from BL is by it made leſs than 360, not by the Double, but the Quadruple of the Square *c* Z.



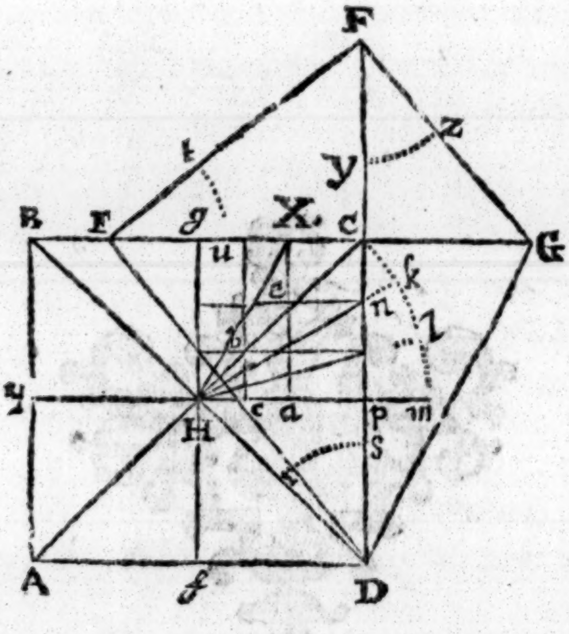
The Cauſe of this Diſagreement is therefore to be enquired into, and it can be no other, but the reckoning of Lines without Latitude in the Proportions of Superficies; whence it follows neceſſarily, that that which is contained within the 4 Sides B L, L Z, Z M, M B, is leſs than 10 Squares from B E, by the Double of *c* Z; but that the Sides themſelves are equal to the Quadruple of *c* Z. Therefore that the Square from B L is the Decuple of the Square from the half Side, ſtands good, and ſhall remain ſo; for it cannot be proved by any Argument, that the Square from B L is leſs than the decupled Square from B E, but that by the ſame it will be proved, that the Square from B C is leſs than 4 Squares of the ſame B E. We have then, by the Solution of this Difficulty, removed one, and perhaps the greateſt Hindrance to Geometry, to wit, the Computation of Superficies by Lines without Latitude.

CHAP. VII.

Of mean Proportionals.

1. HOW to find a mean Proportional betwixt 2 right Lines given, is taught by *Euclid's El.* 6. Prop. 13.

2. To find out 2 mean Proportionals betwixt a right Line given and its Half; let DC be the right Line given, and its Half CG; let them be



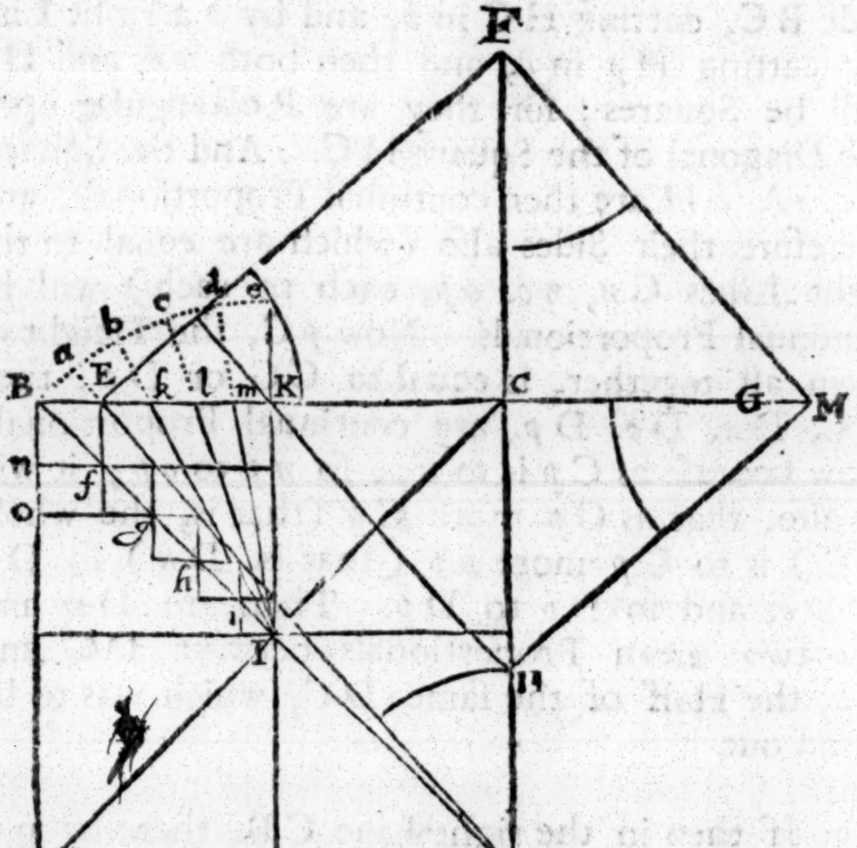
diſpoſed at a right Angle in C; then make a Square DABC, and let it be cut as well by the Diagonals AC, BD, as by the right Lines *g f*, *q p*, meeting 4 Ways in H.

Take the mean Proportional betwixt BC and its Half CG; now that is equal to the Semidiagonal HC, by the Radius HC deſcribe the Arch C*m*, cutting *qp* produced to *m*, and let the Arch C*m* be cut into 3 equal Parts in *k* and *l*, and draw H*k*, H*l* cutting DC in *n* and *o*. Now let C*x* equal to C*n* be taken; draw *xa* parallel and equal to C*p*; cutting HC in *e*, C*e* then is a Square. In like Manner by the Point *o*, draw a Parallel to the Side BC, cutting HC in *b*, and by *b* a right Line *bc* cutting H*p* in *c*, and then both *eb* and H*b* will be Squares; for they are Rectangular upon the Diagonal of the Square HC. And the Squares C*e*, *eb*, *b*H are then continual Proportionals, and therefore their Sides alſo (which are equal to the right Lines C*n*, *no*, *op*, each to each) will be continual Proportionals. Now *p*C, the Height of them all together, is equal to CG or D*p*; then DC, D*n*, D*o*, D*p*, are continual Proportionals. Now becauſe as C*p* is to *np*, ſo *np* to *op*; it will be alſo, that as C*p* more C*p* (that is, the whole DC) is to C*p* more *np* (that is, D*n*) ſo D*n* to D*o*, and ſo D*o* to D*p*. Then are D*n* and D*o* two mean Proportionals betwixt DC and D*p*, the Half of the ſame DC, which was to be found out.

3. If then in the right Line CB, there be put CE equal to D*n*, and CF equal to D*o* be put in DC produced, and FG be joined, and the right Lines DE, EF drawn, the Triangles DEC, ECF, CFG will be Triangles of equal Angles, that is, the Arches *ſr*, *tv*, *zy* (having equal Shanks) will alſo be equal.

4. How to find 4 mean Proportionals betwixt a right Line given, and its Half.

Let the right Line given (in the 2d Figure) be DC, and the Half of it CG; make a Square DC BA, the 4th Part of which is the Square IB. So then the ſtrait Line IB is the mean Proportional betwixt the whole BC and its Half CG.



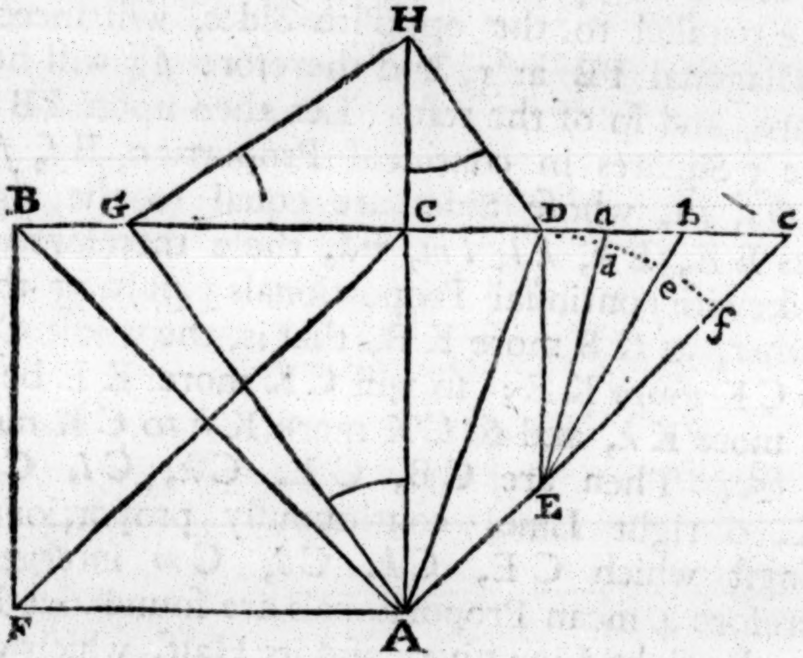
By the Radius IB deſcribe the Arch B*e*, cutting IK produced to *e*; and let it be cut into 5 equal Parts by the ſtrait Lines I*a*, I*b*, I*c*, I*d* cutting

BK in E, *k*, *l*, *m*, and let B *n* equal to BE be taken. The Lines *nf*, E*f* being then drawn, the one parallel to BE, and the other to B *n*, they will meet in the diagonal Line IB at *f*, and B*f* will be a Square. In the ſame Manner, if B *n* be lengthen'd to *o*, that it be equal to B*k*; right Lines drawn by *o* and *k* parallel to the oppoſite Sides, will meet in the Diagonal IB at *g*, and therefore *fg* will be a Square, and ſo of the reſt. Let then upon IB be made 5 Squares in continual Proportion B*f*, *fg*, *gh*, *hi*, *i*I, whoſe Sides are equal to the right Lines BE, E*k*, *kl*, *lm*, *n*I, theſe therefore will be likewiſe continual Proportionals; putting then together, as KB more KB, that is, the whole CB, is to CK more KE; ſo will CK more KE be to CK more K*k*, and ſo CK more K*k* to CK more K*l*, &c. Then are CB, CE, C*k*, C*l*, C*m*, CK, 6 right Lines continuouſly proportional; amongſt which CE, C*k*, C*l*, C*m* intervene. Therefore 4 mean Proportionals are found out betwixt the right Line given and its Half, which was to be done.

5. Conſectary. If in the right Line DC produced, CF equal to C*k* be put; and C*m* equal to C*l* be put in BC produced, and in CD there be put CH equal to C*m*: And laſtly, if HK, HM, MF, FE, ED be joined, there will be made 5 equiangled Triangles, DEC, CEF, FMC, MHC, HKC.

6. How to find 2 mean Proportionals between 2 ſtrait Lines whatſoever given.

Let (in this 3 Fig.) B C the greater, and C D the leſſer, be given. Let them be diſpoſed at right Angles in C, and let B C be equal to A C, let A D, A B be joined, and C *c* equal to the right Line B C be taken, and A *c* joined.



From the Point D, draw down D E, making right Angles at D, and cutting A *c* in E: From the Centre E at the Diſtance E D, deſcribe an Arch of a Circle D *f*, ending in A *c* at *f*; let the Arch D *f* be cut into 3 Parts in *d* and *e*, and the Lines E *d*, E *e* being drawn, produce them to D *c* in *a* and *b*. According then to the 2 Art. of this Chap. *c b*, *b a*, *a* D, will be continual Proportionals; and (by El. 6. Prop. 1.) the Triangles A C D, A D *c*, will be as the Baſes C D, D *c*; and the Triangles A C *c*, E D *c* alike; and the Triangle A C *c*, will be to the Triangle E D *c* in the double Proportion of C *c* to D *c*; therefore C *c*, C *b*, C *a*, C D, will be continual Proportionals.

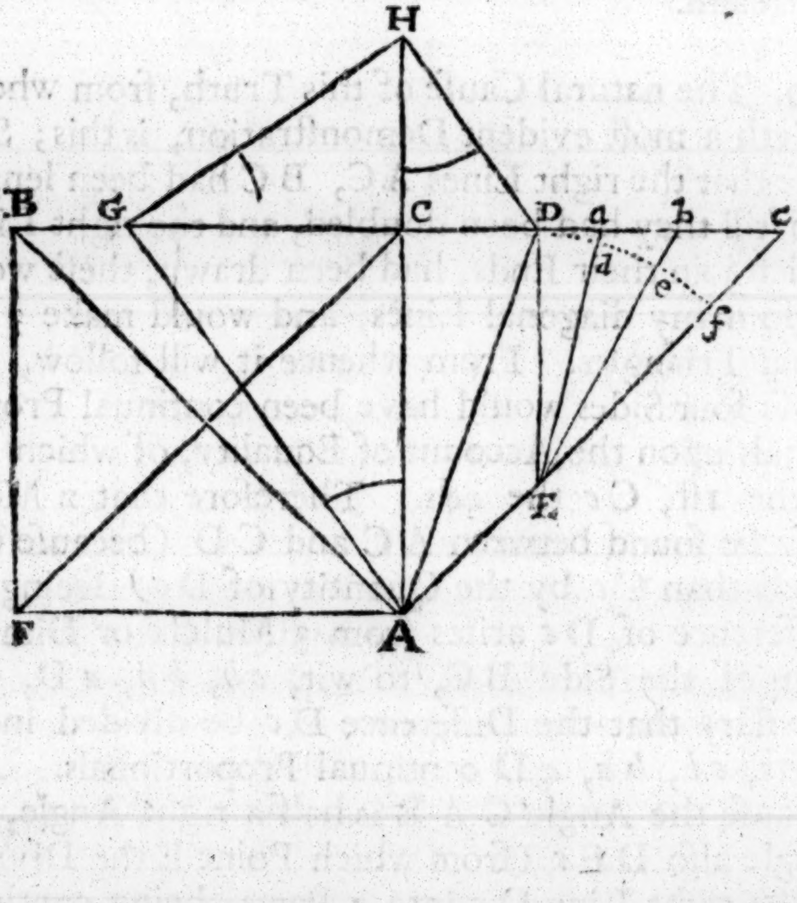
7. Conſectary. If then in the right Line C B, there be put C G equal to C*b*, and in A C produced C H equal to C*a* be put; H D being drawn, and A G, G H joined, the Triangles A C G, G C H, H C D, will have equal Angles.

8. In like manner, if C D were put in the Side C B, and the Arch D *f* cut in five Parts, between A C and C D, four mean Proportionals would be found; but if C D were put in A C produced, and the Angle D E *c* cut in ſeven Parts, ſix mean Proportionals would be found betwixt the ſame Extreams.

9. The natural Cauſe of this Truth, from whence ariſeth a moſt evident Demonſtration, is this; Suppoſe that the right Lines A C, B C had been lengthned 'till they had been doubled, and the right Lines, which join their Ends, had been drawn, theſe would be ſo many diagonal Lines, and would make 3 like equal Triangles. From whence it will follow, that thoſe four Sides would have been continual Proportionals upon the Account of Equality, of which A C is the 1ſt, C *c* the 4th. Therefore that 2 Means may be found between A C and C D (becauſe C D is leſs than C *c* by the Quantity of D *c*) ſeeing the Difference of D *c* ariſes from 3 Mulcts or Diminutions of the Side B C, to wit, *c b*, *b a*, *a* D, it is neceſſary that the Difference D *c* be divided into 3 Parts, *c b*, *b a*, *a* D continual Proportionals. And becauſe the Angle C A B is half a right Angle, the Angle alſo D E *c* (from which Point E the Diviſion of the right Line D *c* into 3 Parts, being continual

Proportionals, taketh its Riſe) ought likewiſe to be half a right Angle.

Now, ſeeing that that Diviſion into 3, is done by a Triſection of a half right Angle, and no otherwiſe; and in taking 4 Means by a Quinquiſection or fivefold Cutting, and no otherwiſe; alſo in taking 6 proportional Means by Septiſection, or a ſevenfold Cutting, and no otherwiſe; and ſo forward: no other Method will ſerve to find out Means that are even in Number, as 2, 4, 6, 8, &c. except the Triſection, Quinquiſection, Septiſection, &c. of a half right Angle.



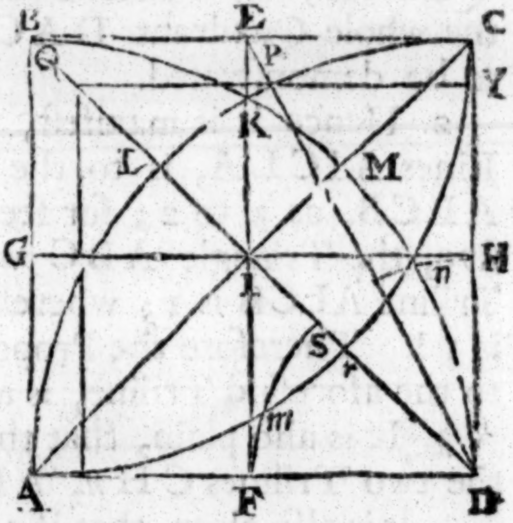
CHAP.

CHAP VIII.

Of the Proportion of a Square, to the Quadrant of a Circle inſcribed in it.

1. LET the Square ABCD be divided, not only by the right Lines, EF, GH, but alſo by the Diagonals AC, BD meeting fourfoldly at I, in the Centre of the Square, and cutting the Arches AC, BD in L and M; and deſcribe from the Centres C and D, the Quadrantal Arches AC, BD mutually cutting one another in the right Line EF at K.

I ſay, that the Square ABCD is to the Quadrant DAC, as 5 to 4.



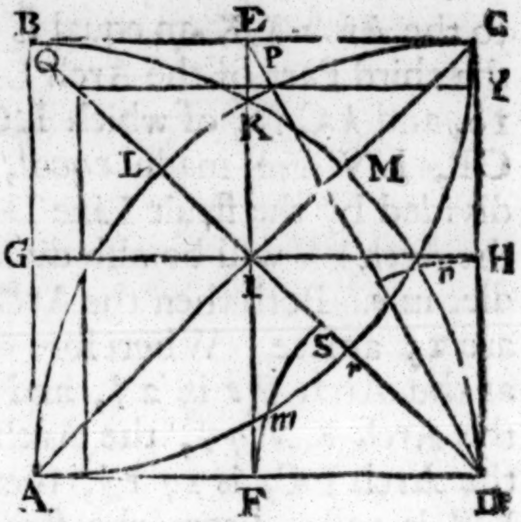
Though I have demonſtrated this in another Place, yet for the ſake of thoſe, who cannot ſo eaſily go along in their Thoughts, with long and difficult Demonſtrations, I ſhall in this Place demonſtrate the ſame by a ſhorter and eaſier Method. By the Radius DH, deſcribe the Arch of a Quadrant HF, cutting the Arch BAC in *m* and *n*, and the Diagonal BD in *s* and *r*; the Quadrant then DHF, is the fourth Part of the Quadrant DCA, and the Space CLF *s* H, is three Fourths of the ſame. Now the ſame Space CLF *s* H, is three Fourths of the Quadrant BA *r* C; taking then that common Space CLF *s* H from both the Qaudrants, there will remain on the one Part the Quadrant DHF, on the other Part the Triline or 3 Lines DA *r* C leſs than the Biline or two Lines *m n*, more than the two three Lines CH *n*, AF *m*, that is, the ſame three Lines DA *r* C, and the Quadrant DHF equal to one another; and therefore the Space BAF *s* HC, will be equal to the Quadrant BA *r* C. For that Space conſiſts of the Triple of the Square DI,. and the fourth Part of the three Lines ALCB, that is, of three Fourths of the Quadrant, and four Fourths of the Triline ALCB. Seeing then the three Lines ALCB, are equal to the fourth Part of a Quadrant; the whole Square ABCD will be to the whole Quadrant DAC, as 5 to 4, which was to be demonſtrated.

2. Hence it is manifeſt, that the Biline or two Lines AICLA, is to the Triline or three Lines ALCB, as 3 to 2; for ſeeing the whole Square is five, the Triangle ABC will be $\frac{1}{2}$, of which the Triline ALCB is 1; wherefore the Biline AICLA is 1 $\frac{1}{2}$. Therefore the Proportion of the ſaid Biline to the aforeſaid Triline, is as 3 to 2.

3. It is alſo plain, that the Biline *m n* is equal to the two Trilines CH *n*, AF *m*.

4. It is alſo clear, that if a right Line were drawn parallel to the Side BC by the Point P, in which the ſtrait Line DE cuts the Arch CL, and ends in the Side DC at Y, and in the Diagonal BD at Q. The Square from YQ would be equal to the Qua-

drant D A C, ſeeing the Square A B C D to the Square from Y Q is as 5 to 4; and the Trilines C Y P, P Q L equal.



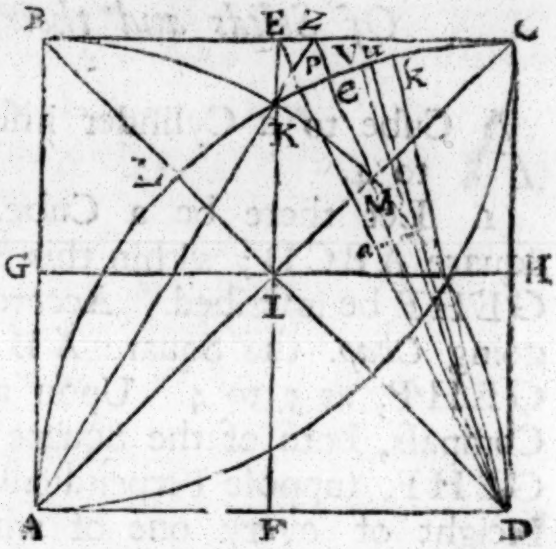
5. It is alſo manifeſt, that the half of the Square A B C D, to wit, the Rectangle AE, the Figure A K F being taken away, is the fifth Part of the Square A B C D: For that which is contained within the three Lines A L K, K B, and B A, together with the Triline B E K, is equal to the Triline ALCB, that is, the fifth Part of the Square A B C D; becauſe B E K, C E K are equal, the ſame alſo is true of the Rectangle D E: Whence it follows, that the two Figures F K D, F K A taken together, are equal to three Fifths of the Square A B C D, that is, to three Fourths of the Quadrant D A C.

Now ſeeing the Knowledge of theſe Things is of it ſelf of no great uſe, I might have paſſed them over; but that, as I had begun to handle Cyclometry, I was willing alſo to perfect it. Likewiſe becauſe to the Fulneſs of Cyclometry, it belongs to know alſo the Quantities of its Parts, that is, of its Segments or Angles; we muſt now treat of theſe.

6. Deſcribe again (in the ſecond Figure) a Square A B C D, divided as in the firſt Figure.

To the Arch C P, let an equal L V be taken, and to the Arch L K an equal C *k*; becauſe then LK is the third Part of the Arch L C; L K will be 10, K *k* 10, and *k* C 10, of which L C is 30. Again, becauſe CP, L V are made equal, the Arch C L being divided by the ſtrait Line D *e* in two in the Middle, the Arch PV will be alſo divided in two in the Middle, in *e*. Both then the Arch C *e*, and the Arch L *e*, are 15 apiece: Wherefore the Arch V *e*, as well as the Arch P *e* is $2\frac{1}{2}$, and the Arch P V 5, and the Arch K V $7\frac{1}{2}$, the Arch L V then, as well as the Arch PC, is 17 1 $\frac{1}{2}$, therefore the Arch C V or KP is $12\frac{1}{2}$. From the ſtrait Line E D, take E *a* equal to the half Side E C, and the Remainder D *a* (according to the 13 *E. Prop.* 1.) will be the greater Segment of the Side D C, or of the ſtrait Line D P divided in extream and mean Proportion. At the Diſtance D *a*, deſcribe an Arch *a b* cutting the Arch D B in *b*; and let D *b* be produced to the Arch C L in *v*; the Arch C *v* then will be 12, of which C L is 30. For (by *El.* 14. *Prop.* 9.) the ſtrait Line D *b* ſubtends the tenth Part of the whole Perimeter, that is a fifth Part of the Semiperimeter, that is, two Fifths of the Arch D B. The Arch then D *b* is two Fifths of the Arch B D; and becauſe the Angle in the Centre, to wit, D A *b* is the double of the Angle in the Circumference, to wit, of the Angle C D *v*, the Arch C *v* will be two Fifths of the Arch C L: Therefore ſeeing C L is 30, C *v* will be 12; now becauſe the Arches L V, C P are equal, C V will be $12\frac{1}{2}$, of which L C is 30: Wherefore as well L V as C P ſhall be $17\frac{1}{2}$ and P *e* or *e* V $2\frac{1}{2}$, and P V or K *e* 5.

If we ſuppoſe the Arch CL to be 45, yet the Proportion of the Angles will be found to be the ſame.



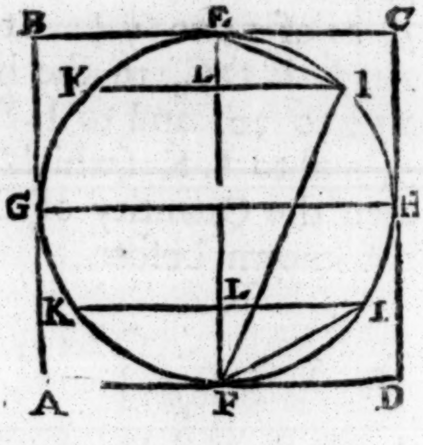
For LK, K *k*, *k* C, will be each of them 15, and L *e*, *e* C each $22\frac{1}{2}$, and K *e* $7\frac{1}{2}$, L P $18\frac{3}{4}$; wherefore C P will be $26\frac{1}{4}$, and LV as much, and C V $18\frac{3}{4}$. Now $18\frac{3}{4}$ more $26\frac{3}{4}$ make 45; but the ſame is the Proportion of 10 to 5, as of 15 to $7\frac{1}{2}$. It ſtands then good, that the Angle E D C, to the half right Angle L D C, is as $17\frac{1}{4}$ to 30, and to L P as $17\frac{1}{2}$ to $12\frac{1}{2}$, or as 7 to 5, and to L K (which is 10.) as 7 to 4. We have then the Quantity of the Angle E D C, that was not known before.

CHAP. IX.

Of Solids and their Superficies.

A Cube to a Cylinder inſcribed in it, is as 5 to 4.

1. Let there be a Cube, whoſe Baſe is the Square ABCD; within this Square let the Circle GEHF be inſcribed. According then to the foregoing *Chap.* the Square ABCD is to the Circle GEHF, as 5 to 4. Upon all the Points of the Compaſs, both of the Square ABCD, and Circle GEHF, ſuppoſe Perpendicular Lines erected, the Height of every one of which is equal to the Height of EF. So a Cube will be deſcribed, together with a Cylinder inſcribed in it, and the Baſe of the Cube will be to the Baſe of the Cylinder, as 5 to 4. Now the Plane that cuts the Cylinder and Cube Parallel-ways to the Baſe, will every where make a Section of the Square, to the Section of the inſcribed Circle, as 5 to 4. Therefore as the Square ABCD to the Circle GEHF (that is, as 5 to 4) ſo (according to *El.* 6. *Prop.* 1.) all the Sections of the Cube together (that is, the Cube) will be (to all the Sections of the Cylinder together, that is, to the Cylinder) as 5 to 4.



2. The Superficie of a Cube, is to the Superficie

of a Cylinder inſcribed in it, as 5 to 4: For the Superficie of a Cube is equal to ſix Squares from the Side ot Height of the Cube. Now *Archimedes*, in his Book of *a Sphere and Cylinder*, hath demonſtrated, that the Superficie of a Cylinder without the Baſes, is equal to the four greateſt Circles in a Sphere, and with the Baſes, to ſix Circles. The whole Superficies then of a Cylinder, is equal to the ſix greateſt Circles in a Sphere: Therefore the Superficie of a Cube, to the whole Superficie of a Cylinder, is as ſix Squares to ſix greateſt Circles, that is, as one Square to one Circle inſcribed in it, that is, as 5 to 4.

3. A Cube to a Sphere inſcribed in it, is as 15 to 8: For a Cube to a Cylinder (as is already ſhewed) is as 5 to 4. Now a Cylinder to a Sphere (as *Archimedes* hath demonſtrated in the firſt Book of a Sphere and Cylinder) is as 3 to 2: Thereſore the Proportion of a Cube to a Sphere, is made up of the Proportions of 5 to 4, and 3 to 2; be it then as 3 to 2, ſo 4 to another; now that ſhall be $2\frac{2}{3}$. The Proportion then of 5 to $2\frac{2}{3}$, is made up of the Proportions of a Cube to a Cylinder, and of a Cylinder to a Sphere: Therefore a Cube to a Sphere is as 5 to $2\frac{2}{3}$, that is, (both being multiplied by three) as 15 to 8.

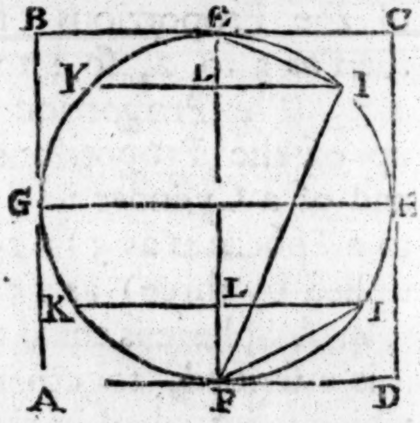
4. A Sphere is equal to the half Cylinder in which it is inſcribed, together with the half of the Cone which is inſcribed in the ſame Cylinder: For ſeeing it is demonſtrated by *Archimedes*, that a Cylinder is to a Sphere inſcribed in it, as 3 to 2, and to the inſcribed Cone, as 3 to 1, a Cylinder, Sphere, and Cone will be as 3, 2 and 1. But 2 (that is, a Sphere) is equal to the Half of the Aggregate of

3 and 1, (that is, to the half of the Cylinder, and half of the Cone put together) as is propoſed.

5. *Archimedes* alſo, in his ſecond Book of the *Sphere and Cylinder*, *Prop.* 1. ſhews how a Sphere equal to a Cylinder may be ſound, to wit, that a Cylinder be taken, which to the Cylinder propoſed is as 3 to 2, that is, as the propoſed Cylinder to a Sphere; then that two mean Proportionals be found betwixt the Height of the taken Cylinder, and the Diameter of the propoſed Sphere: But to find out two mean Proportionals betwixt two right Lines, was not then found out. How that is to be done, I have demonſtrated in the ſeventh Chapter of this Treatiſe.

6. The Superficie of a Sphere is equal to the four greateſt Circles in the ſame Sphere.

For if the Arch of the Semicircle E F be turned round upon the *Axis* E F, it will be the Superficies of the Sphere. From the Point F, let the ſtrait Line F I be how you will drawn to the Circumference, and from the Point I, draw the right Line I K parallel to the Side B C, cutting the Circumference in K, and the Axis in L; the Angles at L will be right; and as F L to L I, ſo will L I be to L E, and as F I is to F L, ſo will E I be to E L, becauſe of the like Triangles F L I, E L I. Now whilſt the Semicircle turns upon the Axis E F, let the Circle I K be deſcribed from the Point I in the Superficie of the Sphere, and it will ſo fall out in



whatſoever Point of the Circumference E F the Point I be placed; but as F E is to L E, ſo is the Circle upon E F to the Circle upon L E; for Circles are in the double Proportion of Rays or Radius's.

Becauſe then all the Perimeters I K make the Superficie of the Sphere, and all the right Lines I K make the Circle GEHF; the Superficie of the Sphere will be to the Circle GEHF in the double Proportion of the Axis E F to the Radius of the Circle GEHF the half of the ſame EF: But the Circle deſcribed on the Axis EF, is alſo to the Circle GEHF in the double Proportion of the Axis E F, to the Semidiameter of the Circle GEHF; therefore the Superficie of the Sphere is equal to the Circle, whoſe Semidiameter is the Axis E F. Now the Circle, whoſe Radius is the Axis of the Sphere, is the Quadruple of the Circle GEHF: wherefore the Superficie of the Sphere is alſo the Quadruple of the ſame.

7. Hence it follows, that the Superficie of any Segment, or Portion of a Sphere, is to the Superficie of the remaining Portion, as the Portion of the Axis cut off by I K is to the remaining Portion; to wit, the Superficie of the Portion FIK, is to the Superficie of the Portion EIK, as the Portion of the Axis F L, to the remaining Portion E L.

8. Laſtly, Becauſe *Archimedes* hath demonſtrated, that the Convex Superficie of a Cylinder is the Quadruple of the Circle, which is the Baſe of the Cylinder, it follows, that it is the ſame thing, as to the Quantity of a Cylinderical Superficie, whether it be made by the Revolution of the Perimeter of the Baſis about the Axis, or by the Motion of the Baſe by the Sides of the Cylinder, or by the circular Motion of the Side. For all theſe beget equal Quantities of Superficie;

ficie; neither is it hard to be underſtood, even without a Demonſtration, that there is no Difference in making a Superficie, whether the Circle that is the Baſe of the Cylinder be carried ſtrait by the Side, or whether in every Point of the Side Perimeters be ſeverally deſcribed, or whether from the whole Side one Perimeter be deſcribed. Now the Superficie of a Cylinder (that is, the thin Coat of the Cylinder ſtript off, its Skin being diſplayed and extended in a Plane) becomes a Triangle, of which one Side is the Diameter of the Baſe, the other a ſtrait Line, whoſe Square is equal to ten Squares from the Diameter of its own Baſis.

CHAP. X.

Of a new Method of treating of Solids, and their Superficies, by the efficient Cauſes.

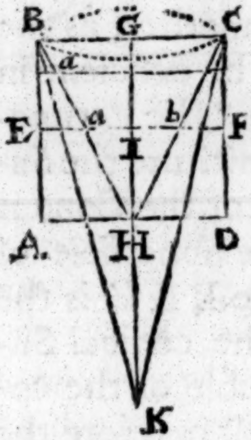
LEMMA.

AN Agent working uniformly in a given Time, for perfecting a deſigned Work, if in ſome Parts of that Time he work, and in ſome others reſt; the Work that is done, will be to the Work left undone, as the Time wherein he worketh, to the Time wherein he reſteth. For Example, If a given Piece of Land may, by continued Labour, be wholly ploughed in three Days; then, if it be only ploughed two Days, and one Day ſpent idly; that which ſhall be ploughed, will be to that which remains unploughed, as 2 to 1; and what was deſigned to be ploughed, will be to what is ploughed, as 3 to 2, and to theu nploughed, as 3 to 1. In like manner,

ner, how much an Agent, by continued and uniform Labour can do in any time, a double Agent, by the like Labour, can do as much, if one half of the Time be ſpent in Work, and the other half idly.

Moreover, how much the Agent loſeth in Time of working, ſo much it muſt be thought he reſted; and theſe are manifeſt by the Light of Nature.

1. The Convex Superficie of a ſtrait Cylinder is equal to the Superficie of a ſtrait Cone, having the ſame Baſe with the Cylinder, but double the Height.

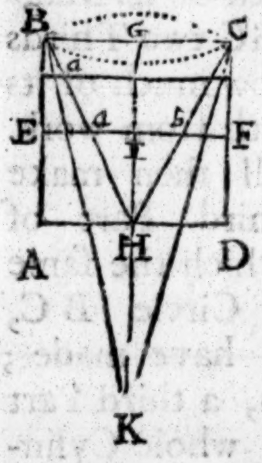


Here (Fig. 1.) take the Cylinder ABCD, ſuch as is propoſed, whoſe Baſe is the Circle BC. Now ABCD is a Square; let the Square ABCD be divided in two in the Middle by the right Line GH, parallel to the Side AB, and lengthen GH to K; let then GK be the Double of GH. Join BK, KC, and BH, CH, then the Square ABCD, and the Triangle BKC have the ſame Baſe, but the Height of the Triangle BKC, is double the Height of the Square ABCD. Suppoſe the right Angle DG to turn round about the Axis GH, and by that Circumvolution from the Rectangle DG a right Cylinder will be deſcribed; but from the Triangles CHG, CKG, two right Cones, and from the Side DC, the Superficie of the Cylinder, and from CH, CK, the Sides of the Cones, two conical Superficies, and from the Point C, the Circumference of the Baſe. Theſe Things ſuppoſed, we are to demonſtrate, that the conical Superficie of the Cone BKC, is equal to the whole Superficie of the Cylinder ABCD.

Draw the right Line *a b* as you pleaſe, but parallel to the Side BC, cutting the ſtrait Lines BH, CH in *a* and *b*. The Proportion then of BC to *a b*, will be the ſame with the Proportion of H *a* to HB; and becauſe *a b* is placed any where, the ſame will the Proportion be every where; and becauſe the right Line BC by an uniform Motion, and to right Angles by the Sides BA, CD, will compleat both the Cylinder ABCD, and its whole Superficie; but the Motion failing according to the Proportions of Times, they will deſcribe the Triangle BHC; the right Line BC reſts as much from working, as it worketh. The Superficie BHC will then be equal to the two Superficies BAH, CDH, that are not made (according to the foregoing Lemma.) Now becauſe the Diameters are in the ſame Proportion one to another, as the Perimeters; the Perimeters alſo will be deficient in the ſame Proportion with the Times. The Perimeters therefore of the Circles, which conſtitute the Superficie BHK, that is, the Superficie of the Cone it ſelf BHC, is equal to the Half of the Superficie of the whole Cylinder. But the Triangle BKC is the double of the Triangle BHC, and the conical Superficie of the Cone BKC, is the double of the conical Superficie of the Cone BHC: Therefore the Superficie of the Cone BKC, is equal to the Convex Superficie of the Cylinder ABCD, which is the Thing propoſed.

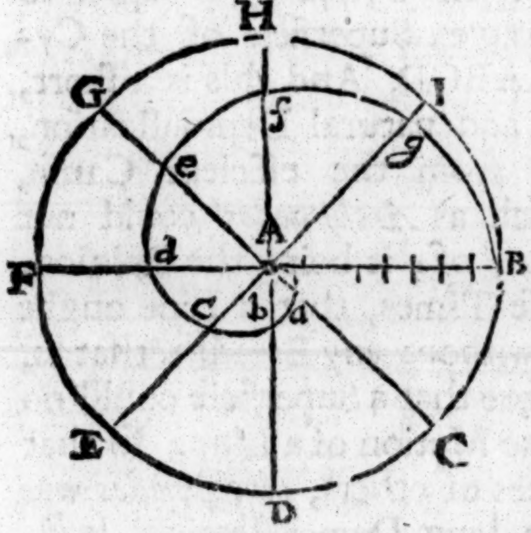
2. The Convex Superficie of a Cylinder, is the Quadruple of the Baſe of the ſame Cylinder; for ſeeing the convex Superficie of a Cylinder is made by the Perimeter of the Circle, which is the Baſe of the Cylinder working uniformly; if a Perimeter, which is the double of the Perimeter of the Baſe, ſhould

work one half of the ſame time, and reſt another half, it will perform as much (according to the foregoing Lemma.) But the Perimeter of the Circle, whoſe Radius is H G, is the double of the Perimeter of the Baſe B C; and the Circle from H G is made by the Radius H G equal to B C, by a circular Motion, that is, the Radius B C ceaſing as much as it operateth, to wit, failing in operating according to the Proportion of the Times wherein it doth operate. Therefore the Circle, whoſe Radius is H G (which is the Quadruple of the Circle that is the Baſe of the Cylinder) is equal to the Convex Superficie of the Cylinder ABCD. And this is a ſhort, clear, and natural Demonſtration, drawn from the efficient Cauſe, but ſuch as *Archimedes* could not make uſe of; it being the Opinion, in thoſe Times, that a Line ought always to be conſider'd without any Breadth, that is, as nothing; and therefore that a Superficie could no ways be deſcribed by the Motion of a Line; ſo that becauſe of the Prejudices of others, *Archimedes* was forced to make uſe of a long Demonſtration, leading *ad impoſſibile*, which no Man's Wit but his own could have overcome.



3. A right Cylinder is the Triple of a right Cone inſcribed in it: For a Cylinder is made by the uniform Motion of the Circle B C (I ſay, of the Circle, not of the Perimeter) which is the common Baſe both of the Cylinder ABCD, and the Cone BHC. Now Circles have a double Proportion of that which their Diameters have: Therefore the Circle BC, to the

Circle *a b*, is in a double Proportion of the Diameter BC to the Diameter *a b*, and ſo every where. Now the Circle, whilſt it maketh the Cone B H C, loſeth every where of its Magnitude in Working, the double Proportion of the Diameter B C to the Diameter *a b*. But (as has been ſhewed, *Chap.* 2. *Art* 9.) when mean Proportionals, as well Geometrical as Arithmetical, are taken every where, all together, they are the ſame; therefore the Circle B C loſeth of its Magnitude, whilſt it maketh the Cone BHC two Thirds of its own intire Magnitude: Now how much of its Magnitude it loſeth, ſo much it ceaſeth from work. It will then make the third Part of that which the ſame entire Circle B C, would have made; that is, a third Part of the whole Cylinder: Therefore a Cone is the third Part of a Cylinder, that is, a Cylinder is the Triple of a Cone inſcribed in it.



4. It may be ſhewed, by the ſame Method, that the ſpiral Space, which ſprings from the firſt Revolution of a Circle, is the third Part of the ſame Circle.

From the Centre A, by the Radius A B, let a Circle be deſcribed, and eight Ways cut by the ſtrait Lines A C, A D, A E, &c. Divide likewiſe the Semidiameter eight Ways, of which let one Eighth A *a* be marked in the ſtrait Line A C, then two Eighths in the Radius A D at *b*, three in the Radius A E at *c*, four in the Radius AF at *d*, five in the Radius AG at *e*, ſix in the Radius AH at *f*, ſeven in the Radius AH at *g*. Now you muſt ſuppoſe that every eighth Part is divided, as the whole Radius, in as many equal Parts, as it can be ſuppoſed it can be divided into; ſo that of what Parts AB is 8, A*a* is 1, A*b* 2, A*c* 3, A*d* 4, A*e* 5, A*f* 6, A*g* 7; then you muſt ſuppoſe a ſpiral Line, (whoſe Beginning is A, and End B) drawn through all the Points, *a*, *b*, *c*, *d*, *e*, *f*, *g*, B; and this will be the ſpiral Line deſcribed by *Archimedes*. We muſt then ſhew, that the Space concluded by this ſpiral Line, and the Semidiameter AB, is the third Part of the Circle BCD——

Becauſe the Circle BCD, is made by the Circumduction of the entire Radius AB, and the ſpiral Space by the ſame Radius, but failing, that is, reſting every where according to the double Proportion of the Times (for all the Circles through *a*, *b*, *c*, *d*, &c. have a double Proportion of the ſtrait Lines A*a*, A*b*, A*c*, &c. the Space which is left without the ſpiral Line unmade, will be the double of that which is made by the ſpiral Converſion; and therefore both the Spaces, made and unmade together, are the Triple of the ſpiral Space.

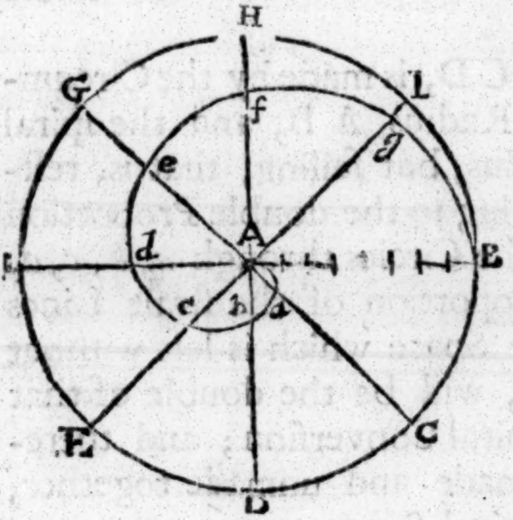
5. Now it is manifeſt, that ſince the ſpiral Line A, *a b*, *c*, *d*, *e*, *f*, *g*, B, is continually deficient in the ſame Proportion with the Times, it is equal to the half Perimeter of the Circle BCD.

6. That a Solid alſo, whoſe Baſe is the ſpiral Space, is the third Part of a Cylinder whoſe Baſe is the Circle BCD.—

7. Alſo that a Square circumſcribed by the Circle BCD—is to the ſpiral Space as 15 to 4. Now (as

we demonſtrated before) a Square is to a Circle inſcribed in it, as alſo a Cube from the Radius, to the Cylinder, as 5 to 4. The Proportion then of a Square to the ſpiral Space, is compounded of the Proportions of 5 to 4, and 3 to 1; ſo that if it be, that as 3 to 1, ſo 4 to another, which ſhall be $\frac{4}{3}$: The Proportion of a Square to the ſpiral Space will be made up of the Proportions 5 to 4, and 4 to $\frac{4}{3}$; the Square then is to the ſpiral Space, as 5 to $\frac{4}{3}$; that is (both being multiplied by 3) as 15 to 4.

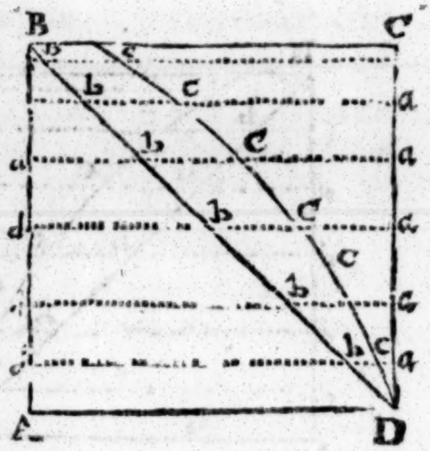
8. It likewiſe follows from the ſame, that if the Circle BCD— be the Baſe of a Cylinder, whoſe Baſe is equal to the Diameter of the ſame Circle, and that there be a Solid, whoſe Baſis is indeed the ſpiral Space, but the Height equal to the Height of the Cylinder; the Sphere wherein the greateſt Circle is BCD, will be the Double of the ſaid Solid. Now we have ſhewed before, that a Square into which the Circle BCD is inſcribed, as alſo a Cube from the Diameter, is to a Cylinder whoſe Baſis is the Circle BCD, as 5 to 4, and to a Sphere, as 15 to 8, and to the ſpiral Space, as 15 to 4; for the Proportion of 15 to 4 being ſubtracted from the Proportion of 15 to 8, there remains $\frac{60}{125}$, that is, $\frac{1}{2}$. The Solid then whoſe Baſis is the ſpiral Space, is the half of the Sphere wherein BCD is the greateſt Circle.



9. By the ſame Method it will be found, that any Rectangle is to the Parabole inſcribed in it, as 3 to 2.

Deſcribe then any right Angle ABCD, in whoſe Side DC take any where D *a*, to which in the Side AB, take an equal A *d*, cutting the Diagonal BD in *b*, and let that be ſo every where; then as *d a* every where to *d c*, ſo let *d c* be to *d b*, and draw the crooked Line B *c c* D through all the Points *c*; that this crooked Line is Parabolical, and the Figure AB *c c* D the half of a Parabole, all Mathematicians do agree. It is then to be ſhewed, that the Rectangle ABCD is to the half Parabole AB *c c* D, as 3 to 2.

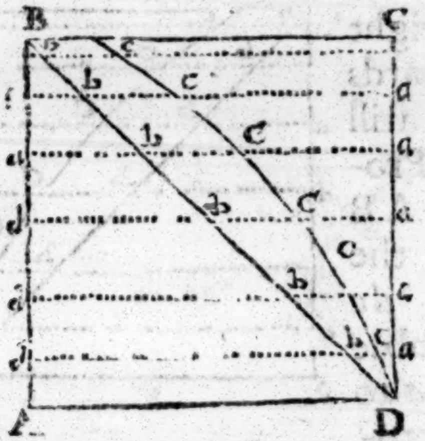
For deſcribe the Rectangle ABCD from the right Line AB uniformly, and parallelways moved to the oppoſite Side DC. Now the Semiparabole AB *c c* D is deſcribed by the right Line CB moved towards the oppoſite Side AB, ſtill loſing in the double Proportion of the time AB to the time D *a*; for the Proportion of *d a* to *d c* is every where the double of the Proportion of *d a* to *d b*; and therefore (according to Ch. 2. Art. 9.) all the *d a* together, are to all the *d c*, in a double Arithmetical Proportion of all the *da* to all the *db*. So then that which is made uniformly from the whole AB (that is, the whole Rectangle ABCD) to that which is made by the Motion of the Side DC towards AB failing or loſing (that is, reſting



ing from work) according to the ſubduplicated Arithmetical Proportion of Time, will be that which is left unmade, to wit, CD *cc* B one Part of the whole Rectangle ABCD, of which that which is perfected, to wit, the half Parabole AB *cc* D, is two.

And ſo the Rectangle ABCD, is 3 of the ſame Parts: The Rectangle ABCD is therefore to the Semiparabole inſcribed in it, as 3 to 2, and the double Rectangle to the whole Parabole inſcribed in it, as 3 to 2.

10. We may alſo uſe the ſame Method in finding out the Proportion of a right Angle to the Parabolaſter (which is called a Cubical Parabole) for if it be every where that as *da* to *dc*, and *dc* to *db*, ſo *db* to a fourth *de* (ſo that there be four continual Proportionals *da*, *dc*, *db*, *de*) the crooked Line of that Parabolaſter will paſs through all the Points *e*; and the Proportion of the Rectangle ABCD will be to the half of that Parabolaſter, as 4 to 3; and ſo you may proceed to the ſecond, third, &c. Parabolaſter, whoſe Proportions to the Rectangle in which they are inſcribed (as alſo of Cylinders to the Cone and Conoides inſcribed in them) are reduced into the Table of Chap. 17. of my Book *De Corpore*. Such is then the natural Aptitude of the *Lemma* prefixed to this Chapter, to the Demonſtrating of the greateſt Problems of pure Geometry,



Geometry, that he who knows by what Proportion of Proportions (for ſeeing Proportion is a Quantity, there will be Proportions of Proportions, as well as of other Quantities) every compared Figure is framed, he cannot be ignorant of their Proportions one to another. But what (may ſome body ſay) is that pure Geometry? Is that Geometry true which is impure? Let him term it mix'd, not impure. But how can a thing pure, mixed to a thing pure, become impure? But neither muſt that be ſaid; but that then it is mixed, when it is applied to Matter. For it is indeed manifeſt, that if it be not applied to Matter, it is uſeleſs, and no more but mere hard Words. I call that pure Geometry, with which nothing of Arithmetick is blended, but what may alſo agree to a continual Quantity, which is the proper and adequate Subject of Geometry. Therefore the chief things that corrupt Geometry, *are the Surdity of Numbers, Longitude without Latitude, and Latitude without Thickneſs*, and the lately introduced Doctrine, that Fraction is Proportion, as if the half were the Proportion of the half to the whole.

CHAP. XI.

Of DEMONSTRATION.

IN the firſt Place it may be doubted, whether Demonſtration goes before, or comes after Knowledge. Moſt ſay, that it goes before; for a Demonſtration is either ſcientifick or frivolous: It is therefore the efficient Cauſe of Knowledge, and for that reaſon goes before its Effect. It muſt then be granted, that the Maſter's Demonſtration goes before the Knowledge of the Scholar; but no more. But if any Concluſion be demonſtrated by any Man, he muſt firſt have known the Truth of it, before he could demonſtrate it either to himſelf or others. For no Man can demonſtrate, that which he knows not whether it be falſe or true: It is then manifeſt that Knowledge in its own nature goes before Demonſtration.

In the next Place it may be asked (ſince Geometry is very uſeful and ornamental to Mankind) who are chiefly the Men to whom we are indebted for ſo great a Good, the Demonſtrators, or not Demonſtrators. It is certain that long before *Euclid*, many large and artful Fabricks were built, the Tower of *Babylon*, the Pyramids of *Egypt*, the wonderful Walls and Gardens in *Babylon*, the Palace of *Perſia*, and others; alſo a Sun-dial was firſt ſhewed at *Lacedemon*. Theſe Works without doubt required Knowledge; yet the Authors of them demonſtrated nothing, but by natural Logick foreſaw the Reality of their future Works, tho' others afterward were curious to do it, with that purpoſe, that they might excite the Minds of many to the Invention of uſeful Things. The Proportions alſo of the five regular Bodies (as you may read in the old *Greek Epigram)* were found out indeed by *Pythagoras*, tho' *Plato* demonſtrated them, and after him *Euclid El.* 3. *Plato*'s Demonſtration is not extant; ſo that we owe thoſe good Inventions to ſeveral Inventors, whoſe Names (except a few) are now loſt, and not to Demonſtrators. How then is Demonſtration uſeleſs? Not indeed to thoſe who are taught. Now to be taught I think not very laudable, though to teach, provided it be rightly done, and without Hire, is honourable. But doth not a Demonſtration of ſome ample Science, hitherto unknown, deſerve great Thanks? Yes, but he that ſhall do that, is to be reckoned among the Inventors of profitable things. And therefore we are indebted to *Euclid*, who firſt of all taught the World the Method of Demonſtrating, that is, of ſound Reaſoning. All agree that moſt ample Thanks are due to thoſe who firſt adviſed Men to aſſociate, and to unite together under the Obedience of one Supreme Power. The next I think is due to them, who ſhall perſuade Men that they ſhould not violate the Compacts and Agreements that once they have made.

Thirdly, Some may ask what a Demonſtration is: Moſt Men, and not without good Ground uſe to call a Demonſtration, an evident Probation of the Truth, in any dubious Queſtion. Every one thinks that he has enough of Demonſtration, when his Mind does fully acquieſce to thoſe things that are alledged for Probation. But the ancient Philoſophers nurſed up in perpetual Diſputations, as often as there was any Debate concerning the Compariſon of Motions and

Magnitudes, for the moſt part they deſcrib'd Figures, and put before the Eyes of their Diſciples, as if ſhew'd by them to the Sight (that is ſomewhat more as they thought than prov'd) call'd their Arguments ἀποδείξεις, (in *Engliſh*) Demonſtrations. But now when any Man is a little too vehement and poſitive in aſſerting, he affirms that he has demonſtrated.

Now a Demonſtration is, when the Truth of the Concluſion hath its firſt Foundation, in thoſe things which are already known to them, to whom he that proves them, ſpeaks. Now theſe Foundations are Definitions, and beſides Axioms, which are not indeed demonſtrated, yet ought to be true and well underſtood; for from Truths nothing but what is true can be inferred, nor can any thing though never ſo true be known, that is not underſtood.

Fourthly, It may be demanded what the Uſe of Definitions and Axioms is: This is the plain Uſe of Definitions, that he to whom a Demonſtration is made, may underſtand what certain an univerſal Signification of the Word the Demonſtrant would have taken; for the Signification that changeth, deceiveth. Therefore a Definition conduceth to the Underſtanding, and without Underſtanding there is neither any Demonſtration, nor in him that learneth can there be any Knowledge, but from thence unſeaſonable and obſcure Diſtinctions are introduced; ſo that of what is ſaid, nothing maketh Impreſſion on the Imagination. There are indeed, ſome Truths which can no Ways be known by Man, but nothing is demonſtrated that cannot be underſtood by him.

The Uſe of Axioms conſiſts in this, that they abbreviate the too long Series of Demonſtrations, to wit, by removing unneceſſary Demonſtrations. For the

Truth of Axioms ought to appear ſooner, and more clearly than the Means themſelves, by which they are proved.

In every Demonſtration, the Cauſe of the Concluſion ought to be in the Antecedents, by Virtue of which it is inferred; that is, in things before demonſtrated, or known by the Light of Nature. Therefore where the Words cohere and hang together, there will be a Demonſtration; for tho' the Cauſe of the ſubject Matter be not known, yet the Concluſion will retain the Truth of the Principles from whence it is derived. Now Demonſtrations of this kind are eaſy, tho' they little advance Knowledge.

That is the chiefeſt of all Demonſtrations, which is drawn from the Production of the ſubject Matter, according to the Order of Nature; and ſo theſe are the moſt uſeful Definitions for Knowledge, wherein the Generation of the ſubject Matter is explained; that is, by what Motion, what Concourſe of Motions, what Proportions of Motions and Times, all Spaces and Magnitudes are determined.

The next Demonſtration to this, is, when (from the Negation of Truth) ſomething impoſſible is inferred. Now this kind of Demonſtration has its Force from this, that from a Truth nothing but Truth can be deduced.

CHAP. XII.

Of FALLACIES.

THat a Man who underſtands his Words, that is, his own Speech, and ſets upon the Proof of it from true Principles (I mean) natural Definitions, proceeding ſlowly in Mathematical Matters, ſhould be long and often deceived, and being admoniſhed, ſhould perſiſt in his Error, is a thing almoſt impoſſible. Speaking properly, there can be no Error in the Intellect; for to err in the Intellect, is the ſame as not to underſtand. The right Uſe of the Tongue, Feet, and Hands, is not demonſtrated by a Maſter, but by Exerciſe we learn it; and though all our Words almoſt change their Signification, according to the Variety of the things whereof we ſpeak or write; yet we uſe them at home, in the Fields, and in the Market, without any hurt, becauſe it is enough for Civil Society, if one underſtand aright what another ſays.

It is otherwiſe in Philoſophy, where nothing but Truth is ſought after; but eſpecially if Glory attend Invention. It is one thing to walk, another thing to walk upon a Rope; the one is eaſy, and without great Damage, tho' a Man ſhould trip; ſo that Negligence is there pardonable: It is not ſo to a Rope-dancer. Truth walks upon a very ſmall Thread, without that metaphorical Latitude of common Converſation; and eſpecially Mathematical Truth, which unleſs it be poiſed by the Weight of Definitions and Axioms, it tumbles headlong, to the Laughter of Spectators.

The chief and moſt frequent Cauſe then of Fallacies in the Mathematicks, is, that they build their Reaſoning upon Definitions not underſtood, or falſe or ambiguous ones, from which no Truth can be deduced. The *Greek* Philoſophers were of the ſame Mind alſo, when to this Science they gave the Name *Mathematicks*, from the *Greek* Word that ſignifies to underſtand; for commonly when any one, ſpeaking to another, doubted if he was underſtood or not, he asked, μανθάνεις, *Do you underſtand?* To which it was anſwered, μανθάνω, or οὐ μανθάνω; that is, *I do*, or *do not underſtand*. So natural it was to denominate the Mathematicks from the Underſtanding. Another Cauſe of Fallacies is, not to know what Motion is, and its Properties; that is, to be ignorant of the immediate natural Cauſe of all things. Thoſe known Fallacies reckoned up by *Ariſtotle*, by which a Child can hardly be deceived, I purpoſely paſs over, as the greateſt Bane of Geometry. In the firſt Place, I condemn a *Line without Latitude*, a thing unconceivable. Secondly, The Side of a ſquare Figure, ſuppoſed for the Root of a Number. Thirdly, The nature of Proportion not underſtood. Fourthly, All conſideration of *infinitum*, whether Geometrical or Arithmetical. And here I ſhould have made an end, had not there been one who affirms, That he can alſo demonſtrate thoſe things, which are neither intelligible, explicable, nor conceivable; which is indeed to ſay, that all Sciences are not worth a Ruſh.

Nothing (ſays he) *is more obvious in Nature than continual Quantity, and local Motion. Now theſe either are, or are not diviſible* in infinitum: *Can this Disjunctive be denied? Can it be ſaid, that they neither are, nor are not thus diviſible? Let any chooſe what Member of this he pleaſes; ſhall he remove the Difficulties that*

that are in it? Or ſhall he anſwer the Objections that can be made to the contrary?

Though theſe be Queſtions, yet in this Place they have the Force of Negations; let him then object his Difficulties. I ſhall explain the Matter itſelf; to wit, that which Mathematical Writers underſtand, when they ſay, that Quantity is diviſible *in infinitum.* They do not ſay nor underſtand, that a finite Quantity (ſuppoſe a Line) is diviſible into Parts that are infinite in Number; but that a Line never ſo little, is of its own nature capable of Diviſion. Neither by Diviſion do they underſtand a material Separation, that is, a Separation of one Part from another; but that in all continual Quantity, a Quantity leſs, and that aſſignable, is ſtill to be ſuppoſed; and that the Signification of this Diviſion is nothing elſe but the unlimitted Conſideration of a Part, in a Whole never ſo little. For of whatſoever it may be ſaid, it is a Whole, it may very well be ſaid, that there are Parts in it; but that a mortal Man can divide any thing eternally, or if he could do it, that yet the Parts ſhould not be of a finite Number, it is impoſſible. Now what is ſaid of a Line, ought likewiſe to be underſtood of Motion, Time, and every thing elſe that is diviſible, except Number. Let an Objection now be brought againſt this (I confeſs I do not remember that ever I read any) that we may ſee, if it may not eaſily be underſtood, whether it be ſtrong or weak. Moreover,

I ſuppoſe (ſays he) *that moſt know that famous Argument of* Zeno, *which is called* Achilles, *and that how that great Diſputer, whilſt that, by apparent Impoſſibilties and Abſurdities on each Side, he demonſtrated local Motion to be impoſſible, he was confuted by one of the Hearers, who roſe and walked through the School.*

Of which Words this is the Sum, That *Zeno* was, indeed, confuted by the Walker, but that his Sentence was rightly inferred by a Demonſtration leading to an Abſurdity. What *Zeno*'s Argument is, that the Reader may judge of it, I will ſhew. Seeing the leaſt Space that is, cannot be paſſed over by Motion, but that the half of that Space is firſt to be paſſed over, and that the leaſt Space again has alſo its half, and ſo perpetually; *Zeno* concludes (ſubſuming that no Space can be paſſed over in an Inſtant) that it requires an infinite Time to paſs over the leaſt Space whatſoever. Now it is manifeſt, that if a Space cannot be paſſed over without an eternal Motion, it cannot at all be paſſed over.

The Stoicks exemplified this Argument taken from *Zeno* their Maſter, in *Achilles* ſwift of Foot, and a ſlow-paced Tortoiſe; and they ſaid (putting *Achilles* and the Tortoiſe into the Race) that *Achilles* could never overtake it, if it were but the leaſt Diſtance before him. But why? becauſe whilſt *Achilles* ran over that half Diſtance, the Tortoiſe alſo advanced a little. I ſhould be aſhamed to repeat thoſe Trifles, did I not perceive, that they who leaſt ought to be, may yet be deceived by ſuch childiſh Fallacies: But what is to be anſwered, I ſhall anſwer, Firſt, That it is no wonder, if he that is unwilling, never overtake a ſlow Runner that is got before him. For ſuch is the nature of Motion, that he who always will, or is forced to ſlacken his Swiftneſs in proportion to the Space that is left, will never be able to paſs over even the ſmalleſt Space whatſoever. Secondly, What is made by any whole, and the half of it, and the half of that half, and ſo on, will be leſs than two; they are ſo far from making an infinite Time. By that Argument of *Zeno*'s then, it cannot be inferred, that ſwift *Achilles* could not in running overtake the Tortoiſe; but only this, that if he pleaſed he would not. Therefore the Example of *Zeno's* Sophiſm does not prove, that there can be a Demonſtration of a thing that is not intelligible. He goes on;

The pure and moſt ſimple Mathematicks are the moſt eaſy, evident, and comprehenſible of all human Sciences. Yet in Geometry and Arithmetick, how many Propoſitions are there firmly demonſtrated, which nevertheleſs are inexplicable, unconceiveable, and incomprehenſible? I ſhall give a few Inſtances, &c.

Is not this abſurd? Can he be ſaid to demonſtrate a Truth, that does not make it apparent and intelligible? How can I know whether it be true or falſe, if I cannot imagine it in my Mind? But let us read the Examples, which he ſays he will ſubjoin.

1. *That the leaſt Space imaginable may be equal to another Space upon the ſame Baſe, and of the ſame Height, whoſe Sides are drawn out* in infinitum.

That a Parallelogram and Triangles, indeed, of the ſame height, upon the ſame Baſe, are all equal to one another, is intelligible to every one. What he adds, *whoſe Sides* are drawn out *in infinitum*, is abſurd; as a Bench is not called a Bench before it be perfected, ſo neither is a Space before it be finiſhed.

For it is downright impoſſible and abſurd; nor from *Torricellius* does it follow, that a Finite is equal to an Infinite.

2. *One Infinite may be greater than another Infinite.* Who ſaid ſo? He is ſilent; for it is abſurd.

3. *That all the circular Angles of Contact are equal.* What is a circular Angle of Contact? The Expreſſion itſelf is unintelligible.

4. *The eternal Appropinquation of two Lines, but the Concourſe impoſſible, to wit, Aſymptotes.* The Cauſe of this is before explain'd in the Sophiſm of *Zeno.*

5. *The Affections of ſurd and irrational Quantites; all theſe are ſo demonſtrated, as that they cannot be denied, and many others, which nevertheleſs are inexplicable, incomprehenſible, and inconceivable.* But what are theſe ſurd and irrational Quantities? That there are ſome continual Quantities, which have not the Proportion of Number to Number, and are called incommenſurable, there is no Mathematician but knows; and of theſe alſo that are commenſurable one with another, many are irrational; becauſe though they be commenſurable one with another, yet becauſe they are not commenſurable to any Quantity taken at Pleaſure, they are ſaid to be irrational.

6. *In Numbers can the Affections of a Unity and Ternary be fully comprehended, expreſſed and explained, that in Nature there ſhould be one Quantity, and no other, which with its infinite Powers aſcending, and Roots deſcending, all ſhould be equal among themſelves, or rather one and the ſame?* The Words themſelves are pretty obſcure; but I think this is it that he would have ſaid: Seeing the ſquare Power of a Number, and a ſquare Number, alſo the ſecond Power and cubical Number, &c. is the ſame thing to Arithmeticians; and that that Number is called the Root of a ſquare Number, which being multiplied by itſelf makes any Number, and the Root of a cubick Number the ſame with that Number, which being multiplied by itſelf, and again by the Product, maketh any one, and that one multiplied by itſelf never ſo often, makes no more but one; it is manifeſt that all the Products and their Roots are the ſame Unities. Now though this Secret be only proper to an Unity, yet

yet it is not hard to be underſtood, becauſe nothing can be multiplied by one. Laſtly,

He adds, *Can ſo much as any one of the infinite potential Roots of the Number 3 be explained, thought, or comprehended?*

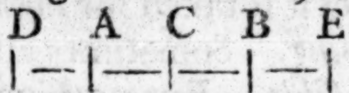
It cannot, I know; and why? Neither that, nor that of many other Numbers, can, I know, and can briefly explain, and that is, becauſe they have no Root.

CHAP. XIII.

Of an INFINITE.

TO the Word *Infinitum* ſomething is underſtood, as Work, Time, Space, *&c.* If it ſignify a Work, then, in its Latin Acceptation, it ſignifies a Work that is indeed begun, but not as yet brought to the End that the Artiſt deſigned in his Mind; and ſo in that Senſe a thing infinite is the ſame as a thing unfiniſhed or imperfect, but what may be finiſhed; ſuch as a Houſe begun, but not perfected. If any then ſhould ſay of an Artiſt, that he had in his Mind to perfect an infinite Work, he would ſpeak abſurdly, as if he ſhould ſay, that it was in his Mind to do that which he never intended: For no Man thinks of doing more than what he can accompliſh; ſo that no Man can judge of a Work that is not his own, without conſulting the Artiſt, whether it be finiſhed and perfect, or infinite, that is, unfiniſhed and imperfect.

An *Infinite*, if Space be underſtood, ſignifies a Space greater than can be equalled by the greateſt Number of Meaſures, as of Feet, Paces, Miles, or even of the Diameters of the Earth, or of the Orb of the fixed Stars; that is to ſay, which cannot be included within Bounds. In like manner, an infinite Time is that which no Number of Hours, or Days can equal.

Therefore of an *Infinite*, according to this Senſe, it cannot be ſaid, that one is greater than another: Draw then the finite right Line A B, and ſuppoſe it produced beyond 

B by E *in infinitum.* Therefore both the right Line B E— and A B E— are infinite in Length. Now A B E— is greater than B E— by the whole length of A B, be it ſo. Let A B be divided in C, and put A D equal to AC; and let it be ſuppoſed lengthen'd ſtraitways by F *in infinitum.* A D F will be then greater than D F by the finite Longitude C D, that is, by the Quantity of A B.

Wherefore C B E— and C A D— are not unequal; therefore there is ſome certain Point, that is a mean in the infinite Line— D B, ſo that the Centre of an infinite Sphere will be the Point C, and (becauſe the Points A and B are taken at Pleaſure) in every Point of an infinite Sphere, will the Centre of the Sphere be; and ſo the Semidiameters of an infinite Sphere from any Centre, whether A, B, or C, are not unequal one to another Therefore one infinite Line is not greater than another.

By the ſame Reaſon, if the Line AB be put for an infinite Time, it may be proved that two Eternals cannot be unequal.

Now the Cauſe why a Finite may be conſidered in an Infinite, conſiſts in this, that neither the ſubject Body is in the Thinker, nor Space, the Image of the Body, in the Thing thought of, but only in the Memory: And in the Memory and Senſe there can be no Infinite.

But Mathematicians uſe often the Word *infinite* for *indefinite*. Now indefinite is the ſame as *never ſo great*, and ſometimes it is taken for *infinitely little*, provided it be not *nothing*. Sometimes alſo *infinite* goes for *as much as is poſſible*. But nothing is properly *infinite*, unleſs it exceed all aſſignable Number of given Meaſures. But it is ſaid to have been demonſtrated by *Torricellius*, that a certain acute hyperbolical Solid is, even in this Senſe of *infinite*, equal to a certain Cylinder, whoſe Baſe hath a Diameter equal to the half Baſe of the Hyperbole, but a Height equal to the tranſverſe Axis of the ſame Hyperbole. I have often and attentively read the Demonſtration of this Problem, and never found any Sophiſm in it. Yet I found that the Diſtance, which *Torricellius* ſuppoſes infinite, is meant of an indefinite Diſtance; nor could it be otherwiſe underſtood by himſelf, who in very many Demonſtrations uſeth the *Cavallerian* Principle of Indiviſibles; which Indiviſibles of *Cavallerius* are ſuch, that their Aggregate may be equal to any given Magnitude. So that ſo abſurd a Propoſition as this, *an Infinite is equal to a Finite*, ought not to be aſcribed to *Torricellius*; for there can be no Solid ſo ſmall, which, being infinite, doth not exceed every finite Solid, as is manifeſt by the

Light of Nature; that is, the Abſurdity of Arithmeticians Reaſoning about an Infinite, and meaſuring Superficies and Solids by Lines without Latitude; who obſerving no Difference betwixt Arithmetick and Geometry, took the Root of a Number (which is part of its ſquare Number) for the ſame thing with the Side of a ſquare Figure, though they confeſs that the Side is no Part of its own Square. So much of Mathematicks. I now expect what the Algebraiſts will ſay to the contrary.

The End of the THIRD BOOK.